

Exclusive weak decays of strange charm mesons from light-cone sum rules

Shan Cheng

Hunan University

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Overview

DiPion

Light-cone distribution amplitudes

Double expansion and the coefficient $B_{nl}^{(l)}(s)$

DiPion(DiKaon) LCDAs in $D_s^{(*)}$ weak decay

$D_s \rightarrow (f_0 \rightarrow) [\pi^+ \pi^-]_S e^+ \nu_e$

D_s^* weak decay

Conclusion

DiPion

Why DiPion ?

- CKM matrix is a crucial criterion of the Standard Model[PDG 2022]
- long standing $|V_{ub}|$ tension
 - † $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$, mainly extracted from $B \rightarrow X_u l \nu$ and $B \rightarrow \pi l \nu$
 - † $|V_{ub}|_{\text{in}} = (4.13 \pm 0.25) \times 10^{-3}$, $|V_{ub}|_{\text{ex}} = (3.70 \pm 0.16) \times 10^{-3}$, $\sim 2.5\sigma$
 - † enlarge the set of exclusive processes to determine $|V_{ub}|$, a candidate is $B \rightarrow \rho l \nu$
 - Δ ρ is reconstructed by $\pi\pi$ invariant mass spectral, width effect/nonresonant contribution ?
 - Δ the underlying consideration is $B \rightarrow \pi\pi l \bar{\nu}_l (B_{l4})$ [Faller 2014]
- V_{cs} issue
 - † $|V_{cs}| = 0.975 \pm 0.006$, mainly extracted from the (semi)leptonic $D_{(s)}$ decays
 - † $|V_{cs}| = 0.972 \pm 0.007$, $|V_{cs}| = 0.984 \pm 0.012$, $\sim 1.5\sigma$ derivation William and Shu-Lei's talks
 - † $\sim 3\sigma$ tension two years ago, 0.939 ± 0.038 and 0.992 ± 0.012
 - † new channels like semileptonic $D_s^{(*)}$ decays are highly anticipated
 - Δ problems encountered, $D_s \rightarrow f_0 l \nu$ has large uncertainty due to the width and complicate structure
 - $\Delta D_s^* \rightarrow \phi l \nu$
- we need to study DiPion and DiKaon LCDAs

- Chiral-even LC expansion with gauge factor $[X, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1) \pi^b(k_2) | \bar{q}_f(zn) \gamma_\mu \tau q_{f'}(0) | 0 \rangle = \kappa_{ab} k_\mu \int dx e^{iuz(k \cdot n)} \Phi_{\parallel}^{ab, ff'}(u, \zeta, k^2)$$

- $\Delta n^2 = 0$, Δ index f, f' respects the (anti-)quark flavor, $\Delta a, b$ indicates the electric charge
- Δ coefficient $\kappa_{+-}/00 = 1$ and $\kappa_{+0} = \sqrt{2}$, $\Delta k = k_1 + k_2$ is the invariant mass of dipion state
- $\Delta \tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2π DAs,
- Δ higher twist $\propto 1$, $\gamma_\mu \gamma_5$ have not been discussed yet, γ_5 vanishes due to P -parity conservation

† Three independent kinematic variables

- Δ momentum fraction z carried by anti-quark with respecting to the total momentum of DiPion state,
- Δ longitudinal momentum fraction carried by one of the pions $\zeta = k_1^+ / k^+, 2q \cdot \bar{k} (\propto 2\zeta - 1)$ Δk^2

† Normalization conditions

$$\int_0^1 \Phi_{\parallel}^{I=1(0)}(u, \zeta, k^2) = (2\zeta - 1) F_\pi(k^2)$$

$$\int_0^1 dz (2z - 1) \Phi_{\parallel}^{I=0}(z, \zeta, k^2) = -2M_2^{(\pi)} \zeta (1 - \zeta) F_\pi^{\text{EMT}}(k^2)$$

- $\Delta F_\pi^{\text{em}}(0) = 1$, $\Delta F_\pi^{\text{EMT}}(0) = 1$,
- $\Delta M_2^{(\pi)}$ is the momentum fraction carried by quarks in the pion associated to the usual quark distribution

DiPion LCDAs

- 2π DAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\phi^{I=1}(z, \zeta, k^2, \mu) = 6z(1-z) \sum_{n=0, \text{even}}^{\infty} \sum_{l=1, \text{odd}}^{n+1} B_{n\ell}^{I=1}(k^2, \mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

$$\phi^{I=0}(z, \zeta, k^2, \mu) = 6z(1-z) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{even}}^{n+1} B_{n\ell}^{I=0}(k^2, \mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2, \mu)$ have similar scale dependence as the a_n of π, ρ, f_0 mesons

$$B_{n\ell}(k^2, \mu) = B_{n\ell}(k^2, \mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{[\gamma_n^{(0)} - \gamma_0^{(0)}] / [2\beta_0]}$$

$$\gamma_n^\perp(\parallel, 0) = 8C_F \left(\sum_{k=1}^{n+1} \frac{1}{k} - \frac{3}{4} - \frac{1}{2(n+1)(n+2)} \right)$$

- Watson theorem of π - π scattering amplitudes

△ implies an intuitive way to express the imaginary part of 2π DAs

△ leads to the Omnés solution of N -subtracted dispersion relation for the coefficients

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0) \text{Exp} \left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_\ell^I(s)}{s^N (s - k^2 - i0)} \right]$$

△ 2π DAs in a wide range energies is given by δ_ℓ^I and a few subtraction constants

DiPion LCDAs

- Soft pion theorem relates the chirally even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\parallel, l=1}(0) = a_n^\pi, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\parallel, l=0}(0) = 0$$

- 2π DAs relate to the skewed parton distributions (SPDs) by crossing

△ express the moments of SPDs in terms of $B_{n\ell}(k^2)$ in the forward limit as

$$M_{N=\text{odd}}^\pi = \frac{3}{2} \frac{N+1}{2N+1} B_{N-1, N}^{l=1}(0), \quad M_{N=\text{even}}^\pi = 3 \frac{N+1}{2N+1} B_{N-1, N}^{l=0}(0)$$

- In the vicinity of the resonance, 2π DAs reduce to the DAs of ρ/f_0

△ relation between the a_n^ρ and the coefficients $B_{n\ell}$

$$a_n^\rho = B_{n1}(0) \text{Exp} \left[\sum_{m=1}^{N-1} c_m^{n1} m_\rho^{2m} \right], \quad c_m^{(n1)} = \frac{1}{m!} \frac{d^m}{dk^{2m}} [\ln B_{n1}(0) - \ln B_{01}(0)]$$

△ f_ρ relates to the imaginary part of $B_{n\ell}(m_\rho^2)$ by $\langle \pi(k_1)\pi(k_2)|\rho \rangle = g_{\rho\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha$

$$f_\rho^\parallel = \frac{\sqrt{2} \Gamma_\rho \text{Im} B_{01}^\parallel(m_\rho^2)}{g_{\rho\pi\pi}}, \quad f_\rho^\perp = \frac{\sqrt{2} \Gamma_\rho m_\rho \text{Im} B_{01}^\perp(m_\rho^2)}{g_{\rho\pi\pi} f_{2\pi}^\perp}$$

DiPion LCDAs

- The subtraction constants of $B_{n\ell}(s)$ [Polyakov 1999, SC 2019, 2023]

(nl)	$B_{n\ell}^{\parallel}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\parallel}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$
(01)	1	0	1.46 \rightarrow 1.80	1	0	0.68 \rightarrow 0.60
(21)	-0.113 \rightarrow 0.218	-0.340	0.481	0.113 \rightarrow 0.185	-0.538	-0.153
(23)	0.147 \rightarrow -0.038	0	0.368	0.113 \rightarrow 0.185	0	0.153
(10)	-0.556 \rightarrow -0.300	-	0.413 \rightarrow 0.375	-	-	-
(12)	0.556 \rightarrow 0.300	-	0.413 \rightarrow 0.375	-	-	-

\triangle firstly studied in the effective low-energy theory based on instanton vacuum

- Above discussions are all at leading twist level
- Subleading twist LCDAs are still in lack

DiPion LCDAs in D_s weak decay
DiKaon LCDAs in D_s^* weak decay

$$D_s \rightarrow (f_0 \rightarrow) [\pi\pi]_S e^+ \nu_e$$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons

$$\Delta D_{(s)} \rightarrow a_0 e^+ \nu [\text{BESIII 18, 21}], D^+ \rightarrow f_0 / \sigma e^+ \nu [\text{BESIII 19}], D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu [\text{CLEO 09}]$$

$$\Delta \mathcal{B} \text{ of } D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0, K_S K_S) e^+ \nu [\text{BESIII 22}], D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu \text{ form factor} [\text{BESIII 23}]$$

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

$$\Delta \text{ isospin symmetry expectation } \mathcal{B}(f_0 \rightarrow \pi^+ \pi^-) / \mathcal{B}(f_0 \rightarrow \pi^0 \pi^0) = 2, \quad \text{possible } \rho^0 \text{ pollution}$$

$$\Delta f_+^{f_0}(0) |V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- Theoretical consideration $\frac{d\Gamma(D_s^+ \rightarrow f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2$
- Observed in the $\pi\pi$ invariant mass spectral, improvement with the width effect

$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dsdq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1^2 \beta_\pi(s)}{|m_S^2 - s + i(g_1^2 \beta_\pi(s) + g_2^2 \beta_K(s))|^2}$$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2) \sqrt{\lambda_{D_s}}}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

- $D_s \rightarrow f_0$ ffs to $D_s \rightarrow [\pi\pi]_S$ ffs

[Hambrock 2015, SC 2017,19,20, Descotes-Genon 2019,23] in $B_{(s)}$ cases

$$D_s \rightarrow (f_0 \rightarrow) [\pi\pi]_S e^+ \nu_e$$

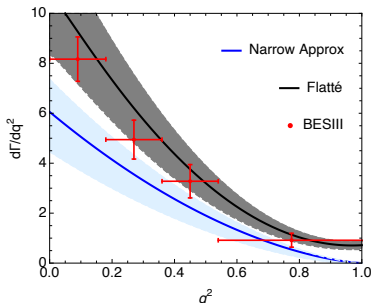
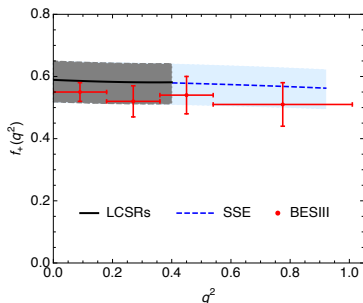
- Definitions of $D_s \rightarrow f_0$ form factors

$$\langle f_0(p_1) | \bar{s} \gamma_\mu \gamma_5 c | D_s^+(p) \rangle = -i \left[f_+(q^2) (p + p_1)_\mu + f_-(q^2) q_\mu \right]$$

- Form factor and the differential decay width [SC 2023]

$\Delta M^2 = 5.0 \pm 0.5 \text{ GeV}^2$ and $s_0 = 6.0 \pm 0.5 \text{ GeV}^2$, $\Delta \tilde{f}_{f_0} = 335 \text{ MeV}$, much larger than 180 MeV used in the previous LCSR, $\Delta a_1^{5/\sigma}$ term contributions are considered for the first time, Δf_0 is not a pure $\bar{s}s$ state, the mixing angle is chosen at $20^\circ \pm 10^\circ$

LCSRs-S1	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

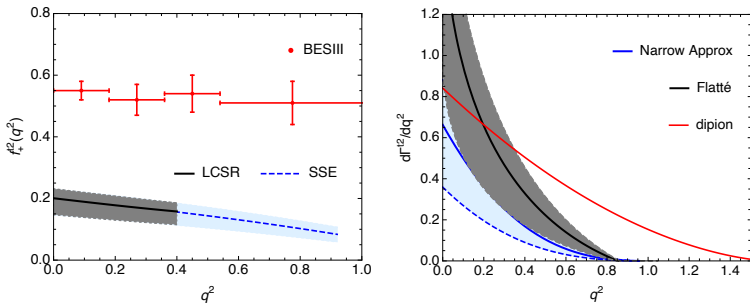


$$D_s \rightarrow [\pi\pi]_S e^+ \nu_e$$

- Definitions of $D_s \rightarrow [\pi\pi]_S$ form factors

$$\langle [\pi(k_1)\pi(k_2)]_S | \bar{s}\gamma_\mu(1 - \gamma_5)c | D_s^+(\rho) \rangle = -iF_t(q^2, s, \zeta)k_\mu^t - iF_0(q^2, s, \zeta)k_\mu^0 - iF_{\parallel}(q^2, s, \zeta)k_\mu^{\parallel}$$

- Form factor and the differential decay width **at leading twist** [SC 2023]



- subleading twist LCDAs give dominate contribution in $D_s \rightarrow [\pi\pi]_S$ transition
- shows relatively moderate evolution with larger allowed momentum transfer
- further measurements would help us to understand the dipion system, ρ , f_0
- different in $B_{(s)}$ case where the leading twist contribution is dominate

DiPion LCDAs in D_s weak decay
DiKaon LCDAs in D_s^* weak decay

D_s^* weak decay

- $\alpha_s : \alpha : G_F \sim \mathcal{O}(1) : \mathcal{O}(1/137) : \mathcal{O}(10^{-5})$
- very hard to measure weak decay from strong and EM interactions
- the total widths of heavy-light vector mesons are still in lack [PDG 2022]
 - $\Delta \Gamma_{D^{*+}} = 84.3 \pm 1.8 \text{ keV} (\rightarrow D^0 \pi^+, D^+ \pi^0, D^+ \gamma)$
 - $\Delta \Gamma_{D^{*0}} < 2.1 \text{ MeV} (\rightarrow D^0 \pi^0, D^0 \gamma), \quad \Gamma_{D_s^{*+}} < 1.9 \text{ MeV} (\rightarrow D_s^+ \gamma, D_s^+ \pi^0, D_s^+ e^+ e^-)$
 - $\Delta \Gamma_{B^*}, \Gamma_{B_s^*}$ no measurement
- but important to properties and $g_{D_s^* D_s \gamma} \rightarrow$ non-perturbative approaches

	$g_{D^{*+} D^+ \gamma}$ (GeV $^{-1}$)	$g_{D^{*0} D^0 \gamma}$ (GeV $^{-1}$)	$g_{D_s^{*+} D_s^+ \gamma}$ (GeV $^{-1}$)
this work	$-0.15^{+0.11}_{-0.10}$	$1.48^{+0.29}_{-0.27}$	$-0.079^{+0.086}_{-0.078}$
HH χ PT [24]	-0.27 ± 0.05	2.19 ± 0.11	0.041 ± 0.056
HQET+VMD [35]	$-0.29^{+0.19}_{-0.11}$	$1.60^{+0.35}_{-0.45}$	$-0.19^{+0.19}_{-0.08}$
HQET+CQM [71]	$-0.38^{+0.05}_{-0.06}$	1.91 ± 0.09	–
Lattice QCD [32]	-0.2 ± 0.3	2.0 ± 0.6	–
LCSR [21]	-0.50 ± 0.12	1.52 ± 0.25	–
QCDSR [20]	$-0.19^{+0.03}_{-0.02}$	0.62 ± 0.03	-0.20 ± 0.03
RQM [72]	-0.44 ± 0.06	2.15 ± 0.11	-0.19 ± 0.03
experiment [16–18]	-0.47 ± 0.06	1.77 ± 0.03	–

LCSRs, hadronic photon NLO [Li 2020]

LCSRs, LP NLO corrections [Pullin 2021]

$$g_{D_s^* D_s \gamma} = 0.60^{+0.19}_{-0.18}$$

very sensitive to different contributions
(radiative corrections, power corrections)
a benchmark to probe the involved dynamics

- impressive lattice QCD evaluation [HPQCD 2013] $\Gamma_{D_s^{*+}}^{\text{HPQCD}} = 0.070(28) \text{ keV}$
 - Δ the longest-lived charged vector meson
 - Δ encourage us to study the exclusive D_s^* weak decay

D_s^* weak decay

- D_s^* weak decay are highly anticipated to determine $|V_{cs}|$
- leptonic decays, helicity enhanced $D_s^* \rightarrow l\nu$, $|V_{cs}|f_{D_s^*}$

$$\Gamma_{D_s^* \rightarrow l\nu} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^3 \left(1 - \frac{m_l^2}{m_{D_s^*}^2}\right) \left(1 + \frac{m_l^2}{m_{D_s^*}^2}\right) = 2.44 \times 10^{-12} \text{ GeV}$$

$$\Delta \mathcal{B}(D_s^* \rightarrow \mu\nu) = \frac{\Gamma_{D_s^* \rightarrow \mu\nu}}{\Gamma_{D_s^*}} \sim \frac{\Gamma_{D_s \rightarrow \mu\nu}}{\Gamma_{D_s^*}} \frac{2m_{D_s^*}^2}{3m_\mu^2} \sim 2 \times 10^{-5}, \text{ close to the LQCD[HPQCD 2013]}$$

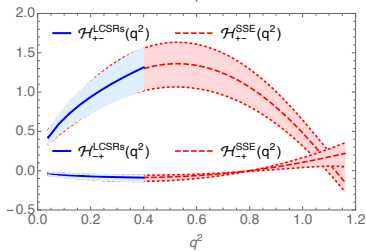
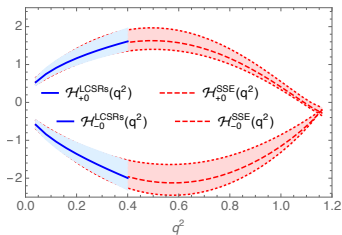
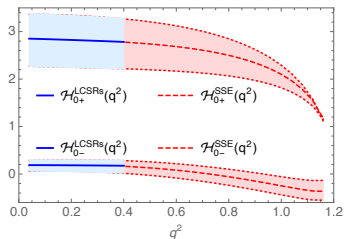
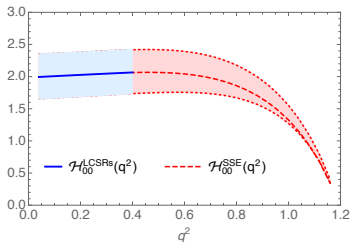
Δ the most favored modes, $(2.1_{-0.9}^{+1.3}) \times 10^{-5}$ [2304.12159 BESIII]

Δ confirms the total width of D_s^* but need more precise lattice evaluation

- semileptonic decays, $D_s^* \rightarrow \phi l\nu$, $|V_{cs}|$ and helicity form factors
 - Δ heavy quark symmetry (HQS) has been examined in $\bar{B} \rightarrow D^*(D)l\bar{\nu}$, also in $D_s^*(D_s) \rightarrow \phi l^+ \nu$?
 - Δ lepton flavour universality (LFU) in vector charm sector
- hadronic decays $D_s^* \rightarrow \phi\rho, \phi\pi$, factorisation theo. or topological analysis
- inclusive decays, $D_s^* \rightarrow X_s l\nu$, HQET and reliability of power expansion

Keri's talk

D_s^* weak decay



- LCSR parameters $s_0 = 6.8 \pm 1.0 \text{ GeV}^2$, $M^2 = 4.50 \pm 1.0 \text{ GeV}^2$
- **Wigner-Eckart theorem:** the helicity information at endpoint is only governed by the Clebsch-Gordan coefficients [Hiller 2014, Grattex 2016, Hiller 2021]

D_s^* weak decay

- semileptonic decays $D_s^* \rightarrow \phi l \nu_l$

$$\frac{d\Gamma_{ij}(q^2)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s^*}^3} \lambda^{1/2}(m_{D_s^*}^2, m_\phi^2, q^2) q^2 |H_{ij}(q^2)|^2$$

$$\Gamma_{D_s^* \rightarrow \phi l \nu_l} = \frac{1}{3} \int_0^{q_0^2} dq^2 \sum_{i,j=0,\pm} \frac{d\Gamma_{ij}(q^2)}{dq^2} = (3.28_{-0.71}^{+0.82}) \times 10^{-14} \text{ GeV}$$

Δ DiKaon LCDAs and the width effect in $D_s^* \rightarrow \phi$ transition

- hadronic decays (naive factorisation)

$$\mathcal{A}(D_s^{*+} \rightarrow \phi \pi^+) = (-i) \frac{G_F}{\sqrt{2}} V_{cs} a_1 m_\pi f_\pi \sum_{i=0,\pm} H_{0j}(m_\pi^2)$$

$$\mathcal{A}(D_s^{*+} \rightarrow \phi \rho^+) = \frac{G_F}{\sqrt{2}} V_{cs} a_1 m_\rho f_\rho^{\parallel(\perp)} \sum_{i,j} H_{ij}(m_\rho^2)$$

$\Delta a_1(\mu) = 0.999$, $f_\pi = 0.130 \text{ GeV}$, $f_\rho^{\parallel} = 0.210 \text{ GeV}$

$\Delta \Gamma_{D_s^{*+} \rightarrow \phi \pi^+} = (3.81_{-1.33}^{+1.52}) \times 10^{-14} \text{ GeV}$, $\Gamma_{D_s^{*+} \rightarrow \phi \rho^+} = (1.16_{-0.39}^{+0.42}) \times 10^{-13} \text{ GeV}$

Δ the result of $\phi\pi$ channel is marginally consistent with the PQCD[Yang 2022]

- with the lattice evaluation of $\Gamma_{D_s^*} = (0.70 \pm 0.28) \times 10^{-8} \text{ GeV}$ [HPQCD 2013]

[SC 2022] $\mathcal{B}(D_s^* \rightarrow l \nu) = (3.49 \pm 1.40) \times 10^{-5}$, $\mathcal{B}(D_s^* \rightarrow \phi l \nu) = (0.47_{-0.10}^{+0.12} \pm 0.19) \times 10^{-6}$

$\mathcal{B}(D_s^{*+} \rightarrow \phi \pi^+) = (0.54_{-0.19}^{+0.22} \pm 0.22) \times 10^{-6}$, $\mathcal{B}(D_s^{*+} \rightarrow \phi \rho^+) = (1.65_{-0.56}^{+0.61} \pm 0.66) \times 10^{-6}$

- Belle II clear background

△ 2022, 400 fb^{-1} , reconstruct 2×10^5 data samples of $D_s^*(D_s)$ from $\phi\pi$ channel

△ phase 3 running (2024-2026), 10 ab^{-1} , $\mathcal{O}(1 \times 10^7)$ data sample of $D_s^*(D_s)$

△ the number of D_s^* production is $\mathcal{O}(10^9) \leftarrow \mathcal{B}(D_s \rightarrow \phi\pi) = (4.5 \pm 0.4)\%$

△ excellent potential to study the D_s^* weak decays, 50 ab^{-1} is hottest expected

- LHCb excellent particle identification to distinguish K, π and μ

△ the channel $D_s^* \rightarrow \phi(KK)\pi$ with the D_s^* producing by $B_s \rightarrow D_s^* \mu \nu$

- BESIII low background

△ directly produced from e^+e^- collision at the $D_s D_s^*$ threshold

△ have collected $\sim 6 \times 10^6$ D_s^* mesons with the 3.2 fb^{-1} data at 4.178 GeV

△ provides the good chance for the leptonic decay $D_s^* \rightarrow l\nu$, Statistical error

△ first \mathcal{B} measurement [2304.12159], determination of spin and parity [2305.14631]

- STCF

Table 2.1: The expected numbers of events per year at different STCF energy points.

CME (GeV)	Lumi (ab^{-1})	Samples	$\sigma(\text{nb})$	No. of Events	Remarks
4.180	1	$D_s^{++}D_s^- + \text{c.c.}$	0.90	9.0×10^8	$\text{BESIII } \mathcal{O}(10^6) D_s^+ / D_s^{*+}$ production Single tag $\text{Belle II } \mathcal{O}(10^9) D_s^+ / D_s^{*+}$ production
		$D_s^{*+}D_s^- + \text{c.c.}$		1.3×10^8	
		$\tau^+\tau^-$	3.6	3.6×10^9	

Conclusion

- DiPion (DiKaon) structure in LCDAs
 - △ width effect of ρ, ϕ, f_0 in CKM determinations △ F_ρ and et.al.
- $D_s^{(*)}$ weak decay
 - △ semileptonic D_s decay provides clean environment to study scale meson and DiPion LCDAs △ D_s^* provides the opportunity of first measurement of weak decay of vector meson and further more physics

Thank you for your patience.