Bayesian determination of the CKM angle γ and the mixing and CP violating parameters entering charm physics

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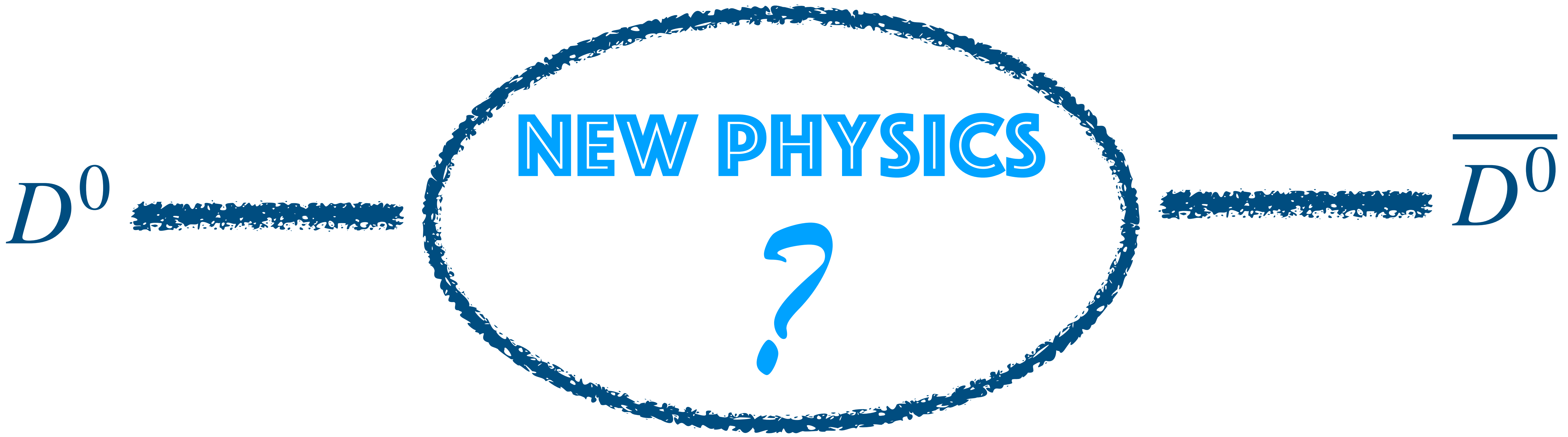
Istituto Nazionale di Fisica Nucleare

Motivations

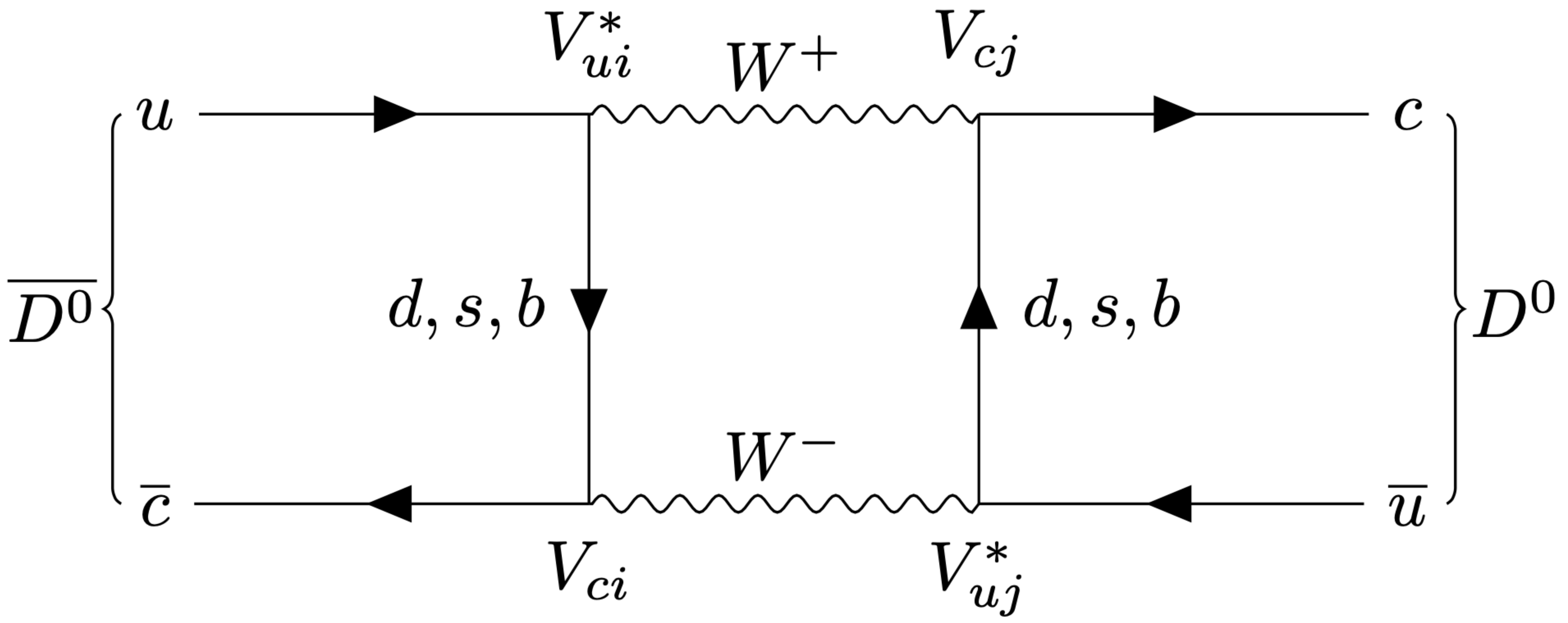
- **FCNC** are absent at the tree-level in the SM and suppressed by the hierarchical structure of the CKM matrix elements and the GIM mechanism.
- **Heavy New Physics** coupled to the up-type quarks **may enter charm mixing**, contributing to the **CP-violating parameters** describing processes involving D mesons.
- The precision reached by modern experiments has made **charm physics** a true **benchmark of the SM**.

How big is this window today?



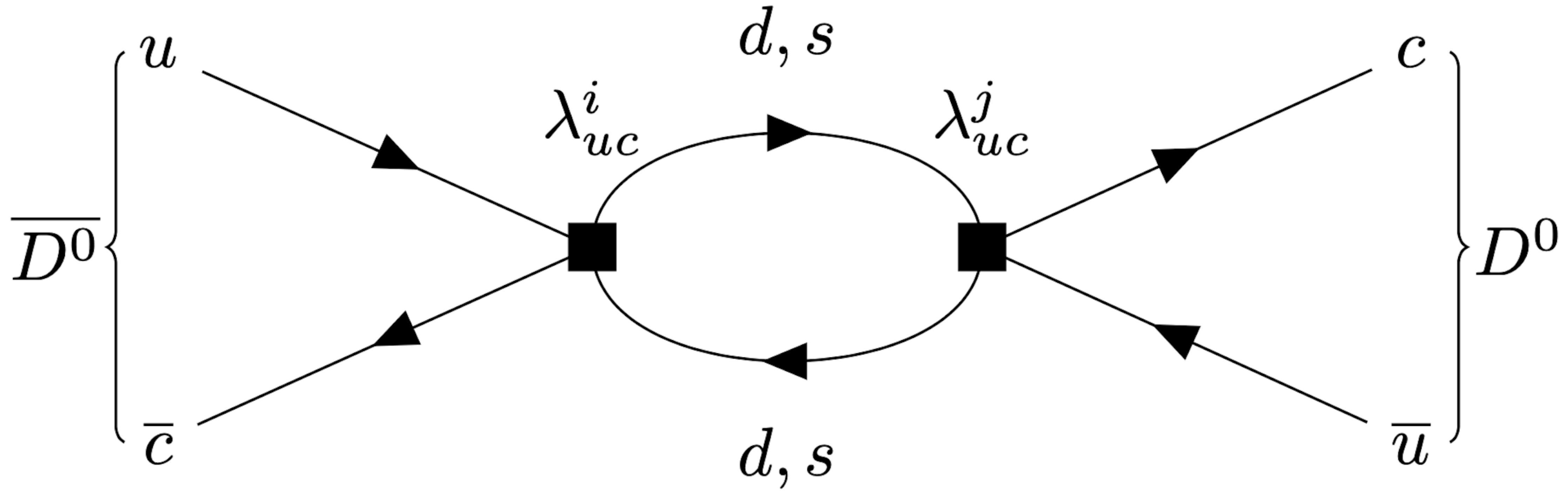
Charm mixing

Short distance



Can be calculated using a local $\Delta C = 2$ Effective Hamiltonian

Long distance



Inherently non perturbative (very hard to compute)

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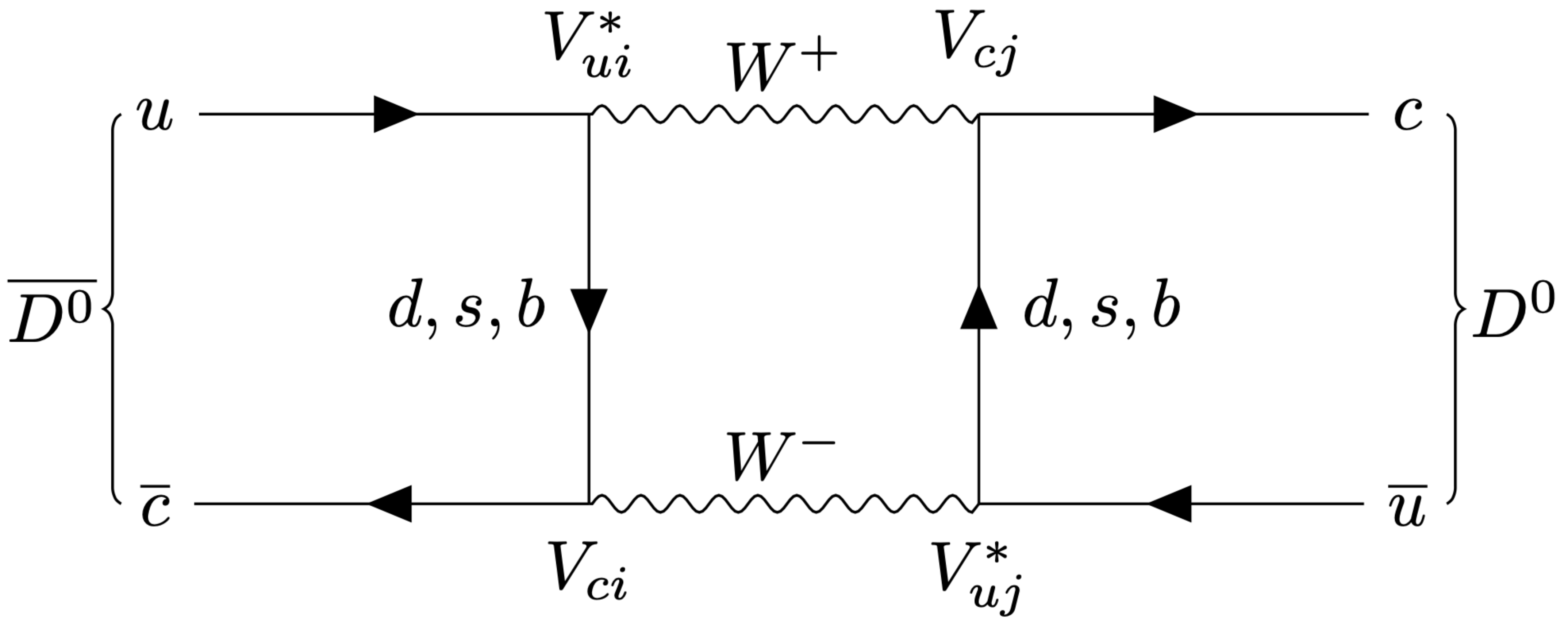

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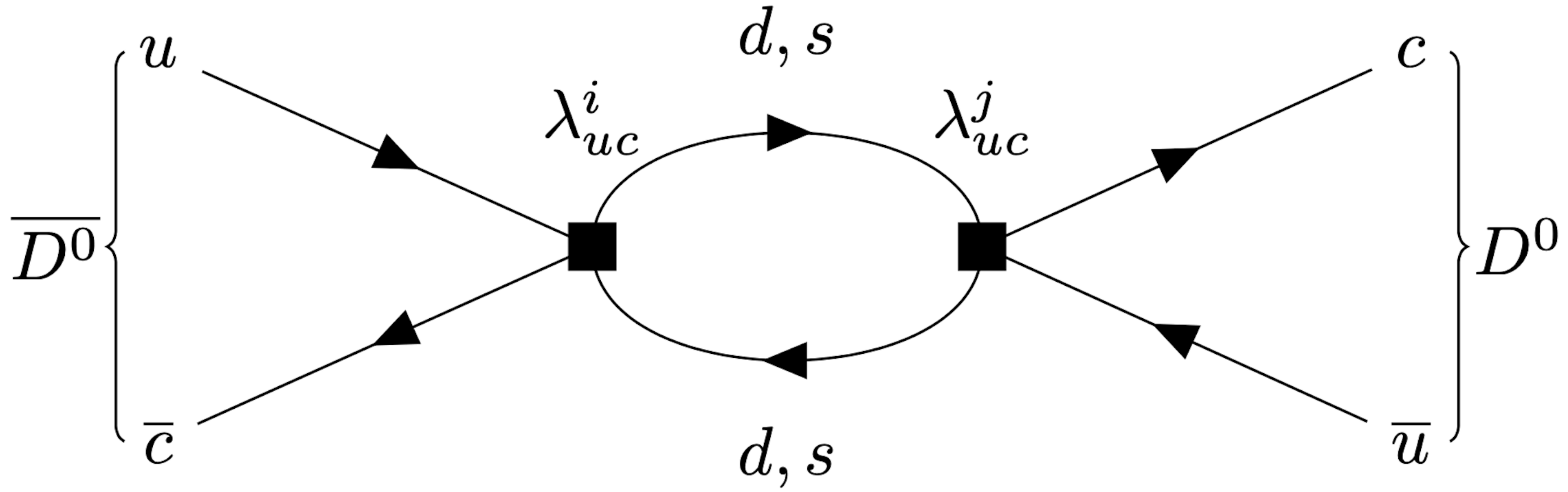

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From GIM + CKM: Long distance dominate the meson-anti meson transition amplitude.

Charm mixing

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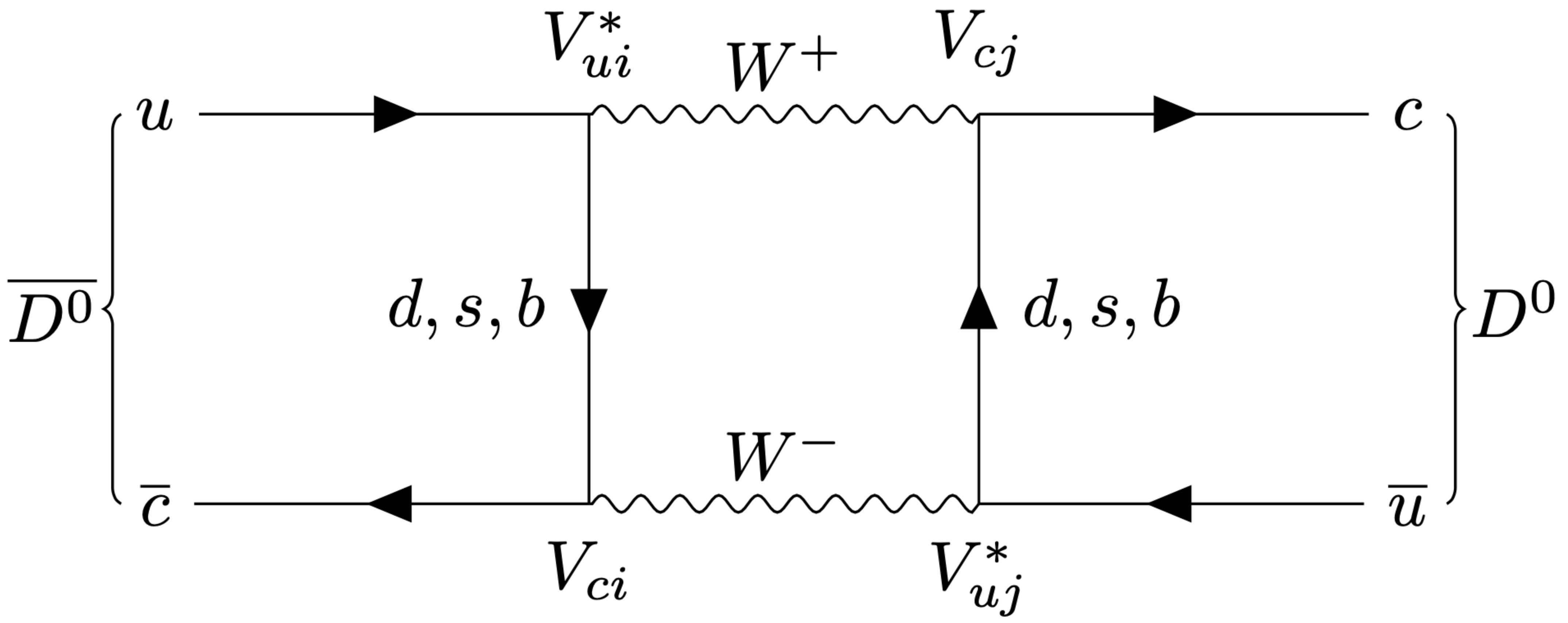

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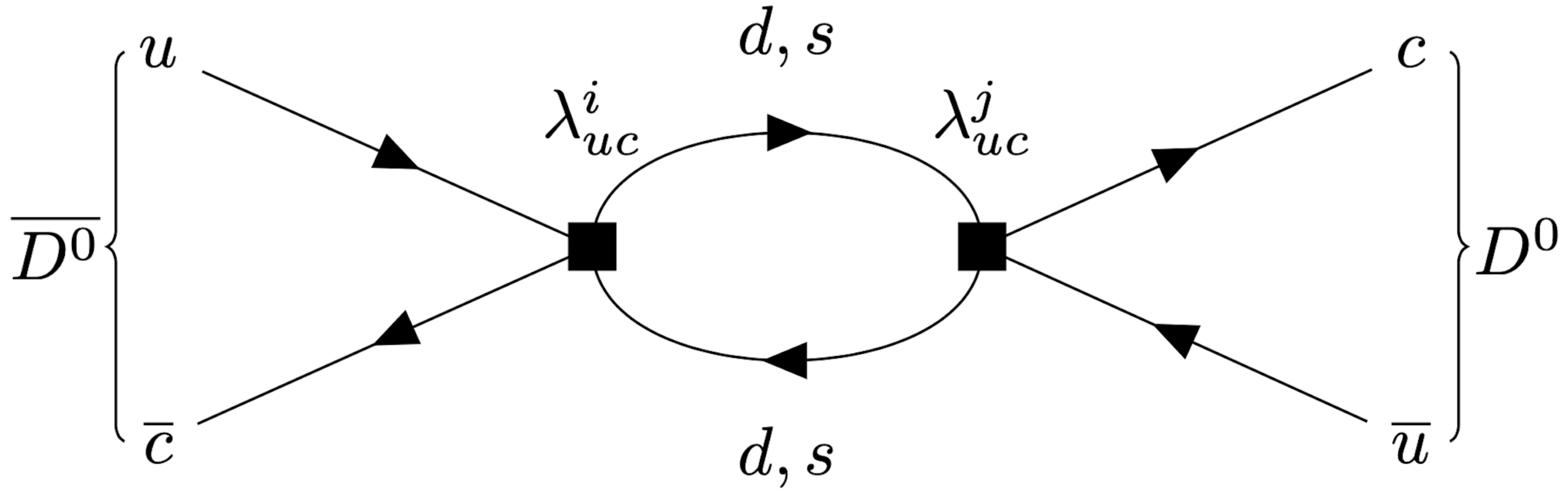

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From the Wolfenstein parametrization of : the CKM

NO Physical phases up to $\mathcal{O}(\lambda^4)$

Large amounts of CP violation could signal the presence of NP

Kagan-Silvestrini parametrization

Hamiltonian formalism for neutral meson mixing

Dispersive Part $H = M - i\,\Gamma/2$ **Absorptive Part**

Hamiltonian eigenstates $|M_{L,S}> = p\,|M^0> \pm q\,|\overline{M^0}>$

Kagan-Silvestrini parametrization

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CP violating parameters A. Kagan, L. Silvestrini 2001.07207

Pure mixing

$$\phi_{12} = \arg\left[\frac{M_{12}}{\Gamma_{12}}\right]$$

Interference between mixing and decay

$\phi_f^{M,\Gamma}$ weak phases of

$$\lambda_f^{x_{12}} = \frac{x_{12}}{|x_{12}|}\frac{\mathcal{A}_f}{\overline{\mathcal{A}}_f}\bigg|_{x=M,\Gamma}$$

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Mixing parameters

$$x_{12} = \frac{2|M_{12}|}{\Gamma} \qquad y_{12} = \frac{|\Gamma_{12}|}{\Gamma}$$

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Pure mixing

$$\phi_{12} = \arg\left[\frac{M_{12}}{\Gamma_{12}}\right] \Longleftrightarrow \left|\frac{q}{p}\right| - 1 \neq 0$$

Interference between mixing and decay

$\phi_f^{M,\Gamma}$ weak phases of

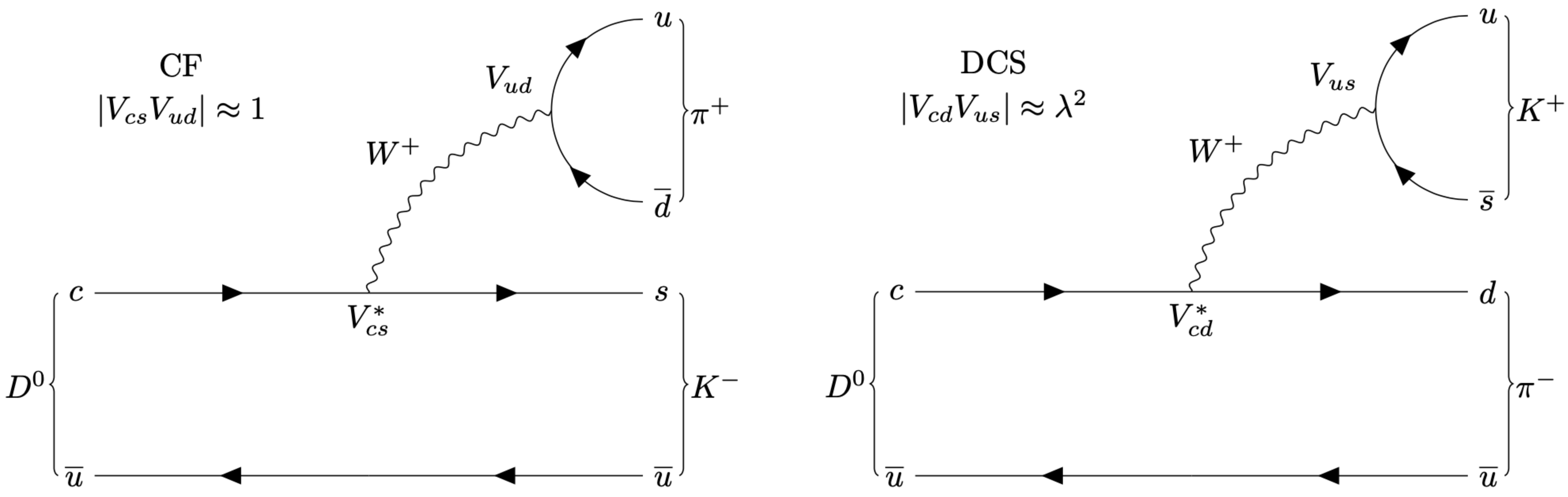
$$\lambda_f^{x_{12}} = \frac{x_{12}}{|x_{12}|}\frac{\mathcal{A}_f}{\overline{\mathcal{A}}_f}\Bigg|_{x=M,\Gamma} \Longleftrightarrow \arg(\lambda_f \lambda_{\bar{f}}) \neq 0$$

Mixing parameters

$$x_{12} = \frac{2|M_{12}|}{\Gamma} \Longleftrightarrow x = \frac{M_S - M_L}{\Gamma}$$

$$y_{12} = \frac{|\Gamma_{12}|}{\Gamma} \Longleftrightarrow y = \frac{\Gamma_S - \Gamma_L}{2\Gamma}$$

Consider the **CF/DCS decays** of the D meson (e.g. $f = K^-\pi^+$)



How to extract ϕ_f^M and ϕ_f^Γ: WS/RS ratios

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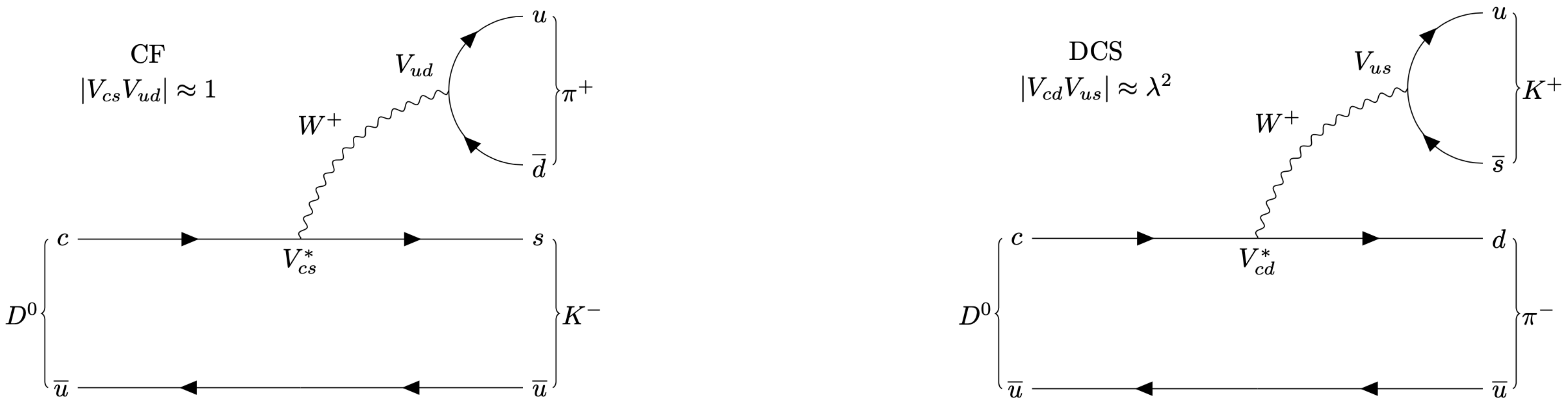


The so-called **WS/RS time-dependent ratios** are measured (e.g. 1611.06143)

WS/RS = **Approximation!!** Second order in x_{12}, y_{12}, since $\mathcal{O}(x_{12}) = \mathcal{O}(y_{12}) \approx 10^{-3}$ = $R_f^\pm + (\Gamma t)\sqrt{R_f^\pm} c_f^\pm + (\Gamma t)^2 c_f'^\pm$

Observables!!

$$R_f^\pm = r_{D[f]}^2 (1 \pm A_D)$$

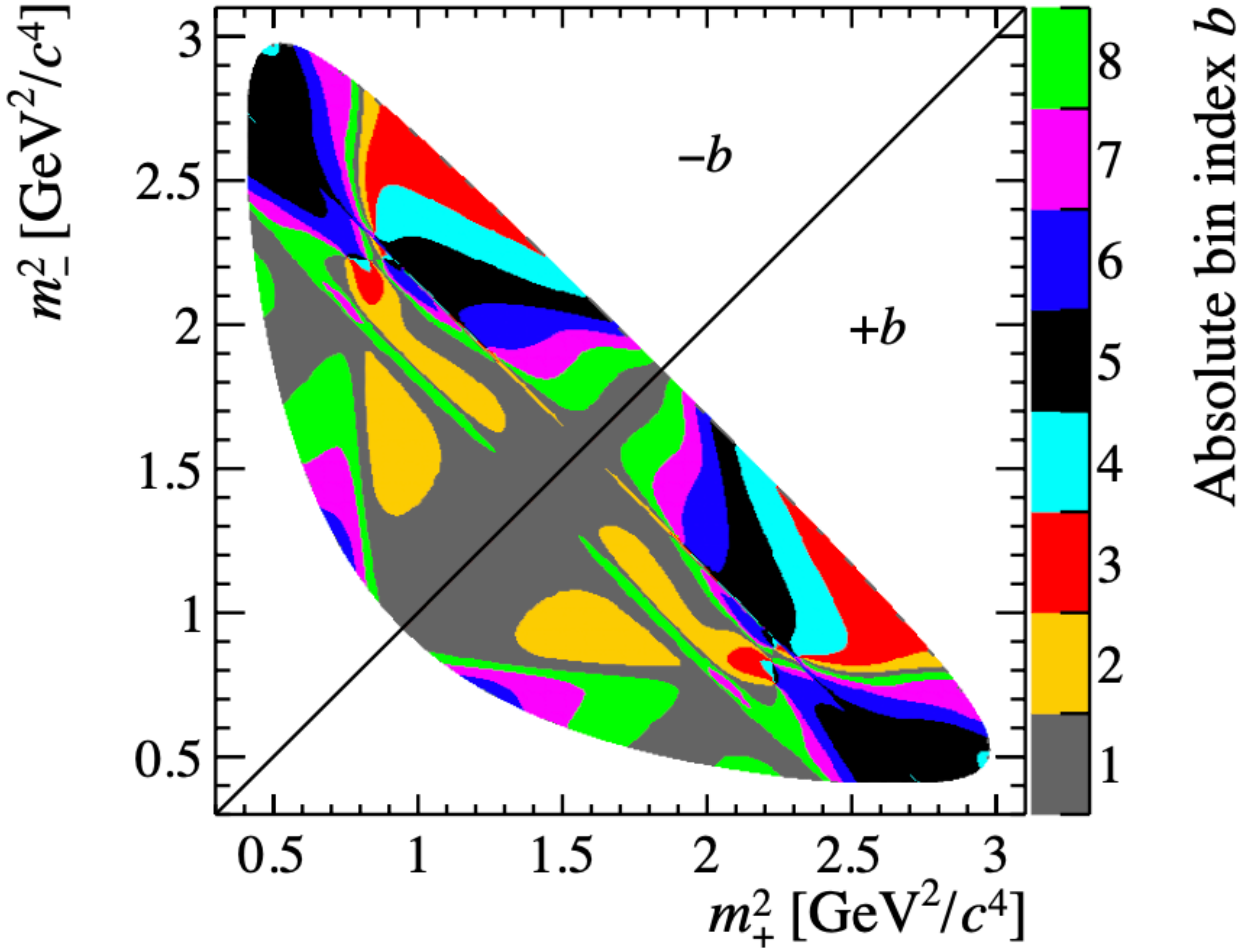
$$c_f^{(\prime)\pm}(x_{12}, y_{12}, \phi_f^M, \phi_f^\Gamma)$$

How to extract ϕ_f^M and ϕ_f^Γ: three-body final states

A study of ratios of decay rates can be performed also for three-body final states (e.g. $f = K_S^0\pi^+\pi^-$) by partitioning the phase space and the decay time in bins (i, j) and counting the relative events

Measuring the Ratios

$$\overset{(-)}{R}_{ij} = \frac{d\Gamma_{\mp ij}\left(\overset{(-)}{D^0} \to f\right)}{d\Gamma_{\pm ij}\left(\overset{(-)}{D^0} \to f\right)}$$



Binning example from 2106.03744

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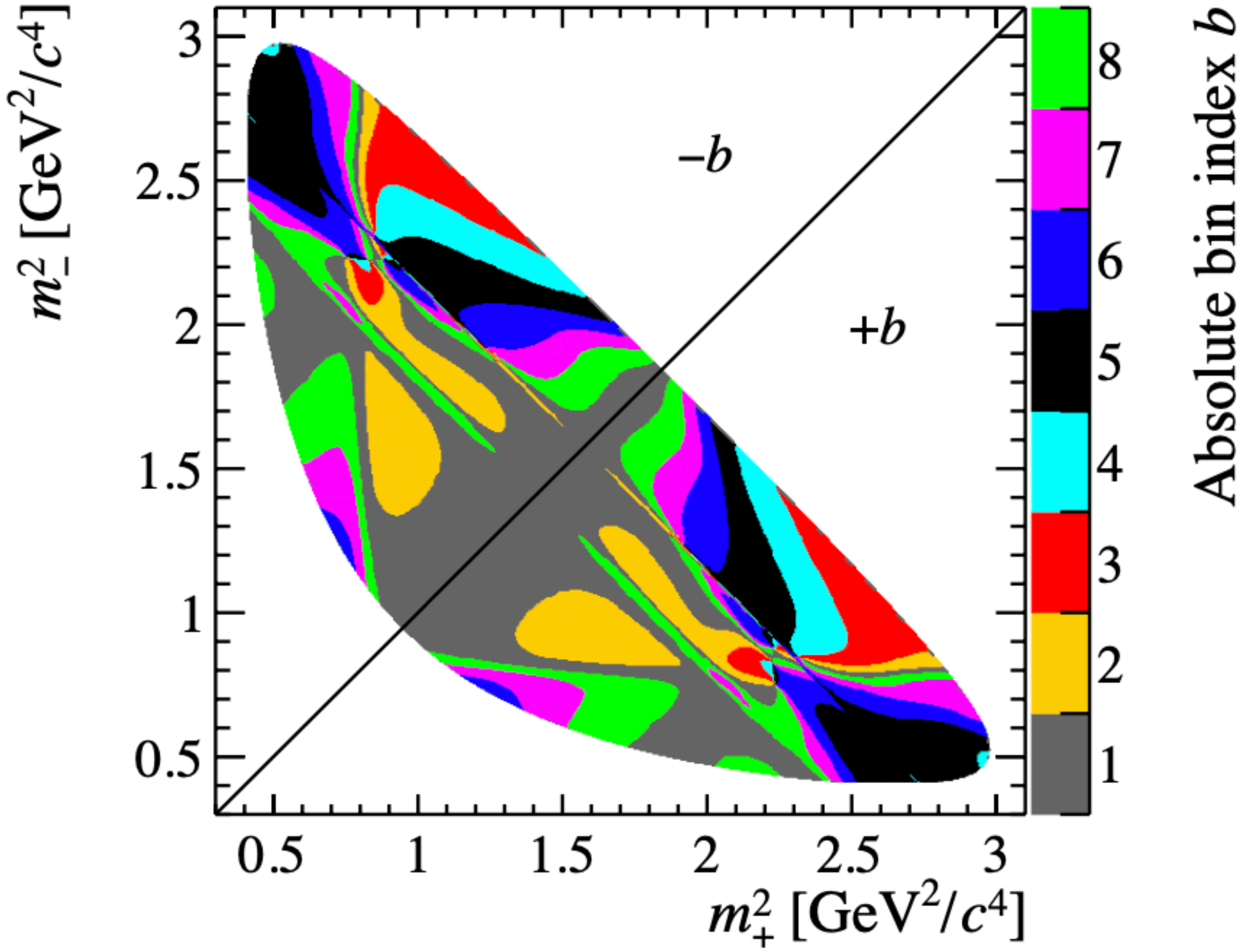
Observables!!

$$x_{CP}^f = x_{12}\cos(\phi_f^M) \qquad y_{CP}^f = y_{12}\cos(\phi_f^\Gamma)$$

$$\Delta x^f = -\,y_{12}\sin(\phi_f^\Gamma) \qquad \Delta y^f = x_{12}\sin(\phi_f^M)$$



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How to extract ϕ_f^M and ϕ_f^Γ: exponential approximation

Other observables rely on a first-order approximation of the decay rate

Exponential approximation: $\Gamma(\overset{(-)}{D} \to f) \propto \exp\left[-\Gamma t\left(\hat{\Gamma}_{\overset{(-)}{D} \to f}\right)\right]$

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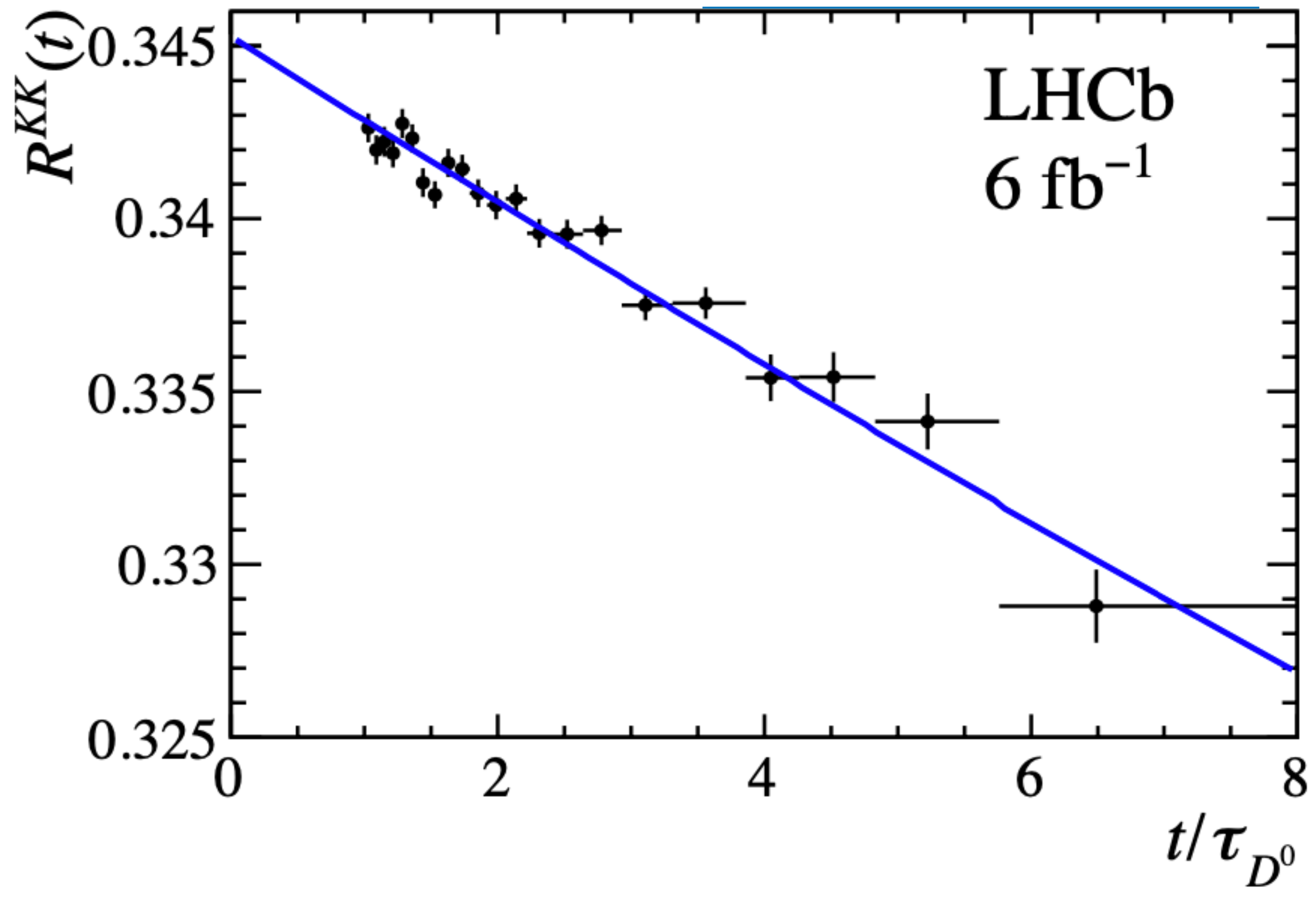
Plot from 2202.09106



Measuring the Ratios

$$R^f(t) = \frac{\Gamma(D^0 \to f_{CP}) + \Gamma(\overline{D^0} \to f_{CP})}{\Gamma(D^0 \to f) + \Gamma(\overline{D^0} \to \bar{f})} \propto e^{-t\Gamma(y_{CP}^{f_{CP}} - y_{CP}^f)}$$

Observables!! $\tilde{y}_{CP}(x_{12}, y_{12}, \phi_f^{M,\Gamma}) = y_{CP}^{f_{CP}} - y_{CP}^f$

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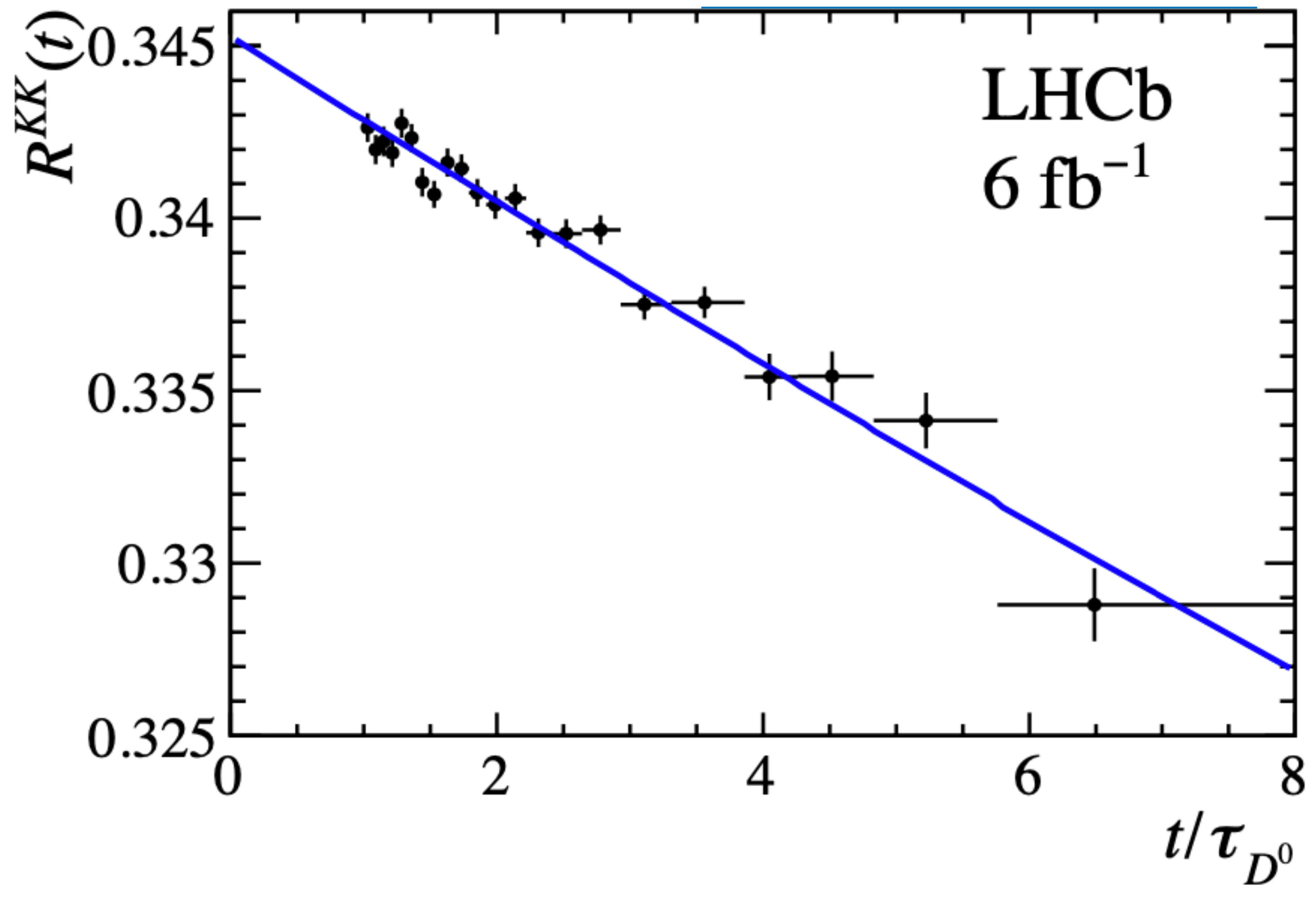

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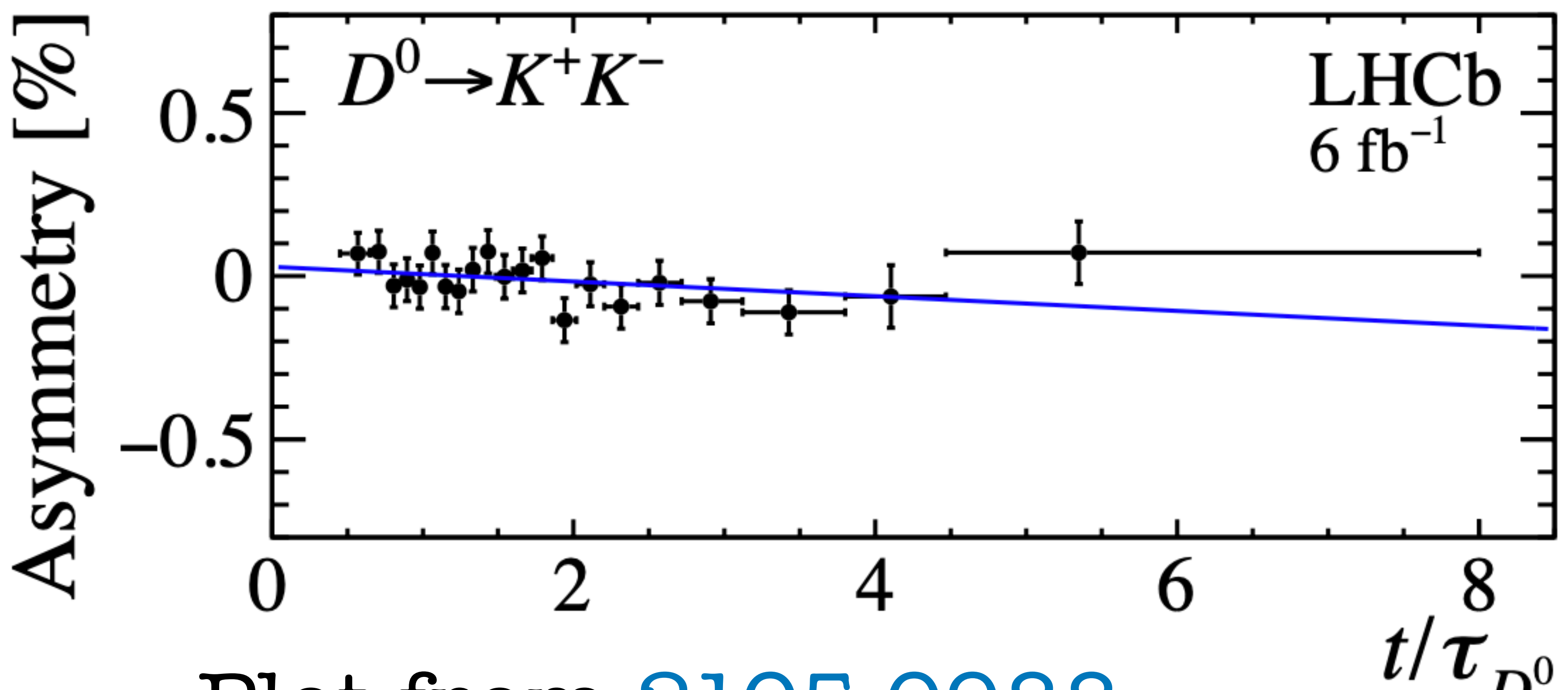
Observables!! $\tilde{y}_{CP}(x_{12}, y_{12}, \phi_f^{M,\Gamma}) = y_{CP}^{f_{CP}} - y_{CP}^f$

Measuring the CP Asymmetries

$$A_{f_{CP}}(t) = \frac{\Gamma(D^0 \to f_{CP}) - \Gamma(\overline{D^0} \to f_{CP})}{\Gamma(D^0 \to f_{CP}) + \Gamma(\overline{D^0} \to f_{CP})} = a_{f_{CP}} + t\Gamma\Delta Y_{f_{CP}}$$



Plot from 2105.0988

Observables!! $\Delta Y_{f_{CP}} = \eta_{f_{CP}}(-x_{12}\sin(\phi_f^M) + a_{f_{CP}} y_{12})$

Subset of charm observables

Obs.	D⁰ decays	Ref.	Obs.	D⁰ decays	Ref.
x_{CP} y_{CP} Δx Δy	$D^0 \to K^0_S\pi^+\pi^-$	[73]	x y	$D^0 \to K^0_S\pi^+\pi^-$	[74]
x_{CP} y_{CP} Δx Δy	$D^0 \to K^0_S\pi^+\pi^-$	[48]	$\frac{x^2+y^2}{4}$	$D^0 \to K3\pi$	[101]
$R^{\pm}_{K\pi}$ $(x'^{\pm}_{K\pi})^2$ $y'^{\pm}_{K\pi}$	$D^0 \to K^{\mp}\pi^{\pm}$	[86]	$F_{D[4\pi]}$	$D^0 \to 4\pi$	[102]
$\frac{\mathcal{B}(D^0\to K^0_s K^+\pi^-)}{\mathcal{B}(D^0\to K^0_s K^-\pi^+)}$ $\Delta_{[K^0_S K\pi]}$ $\kappa_{D[K^0_S K\pi]}$	$D^0 \to K^0_S K^{\mp}\pi^{\pm}$	[93]	$r_{D[K3\pi]}$ $\Delta_{[K3\pi]}$ $\kappa_{D[K3\pi]}$ $r_{D[K\pi\pi^0]}$ $\Delta_{[K\pi\pi^0]}$ $\kappa_{D[K\pi\pi^0]}$	$D^0 \to K3\pi$ $D^0 \to K^{\mp}\pi^{\pm}\pi^0$	[95]
$\frac{\mathcal{B}(D^0\to K^0_s K^+\pi^-)}{\mathcal{B}(D^0\to K^0_s K^-\pi^+)}$	$D^0 \to K^0_S K^{\mp}\pi^{\pm}$	[94]	ΔY	$D^0 \to X^+X^-$	[75]
ΔA_{CP} $\Delta\langle\tau\rangle$	$D^0 \to X^+X^-$	[46]	$\tilde{y}_{CP}$	$D^0 \to K^{\mp}\pi^{\pm}$ $D^0 \to X^+X^-$	[92]
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A pair of weak phases $\phi_f^{M,\Gamma}$ for each of the final states

APPROXIMATE UNIVERSALITY

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Obs.	$\mathbf{D^0}$ decays	Ref.	Obs.	$\mathbf{D^0}$ decays	Ref.
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Decay parameters

They appear when parametrizing the decay amplitudes:

$r_{D[f]}$: Ratios of the magnitudes

$\Delta_{[f]}$: Strong phases

$\kappa_{D[f]}$: Coherence factors

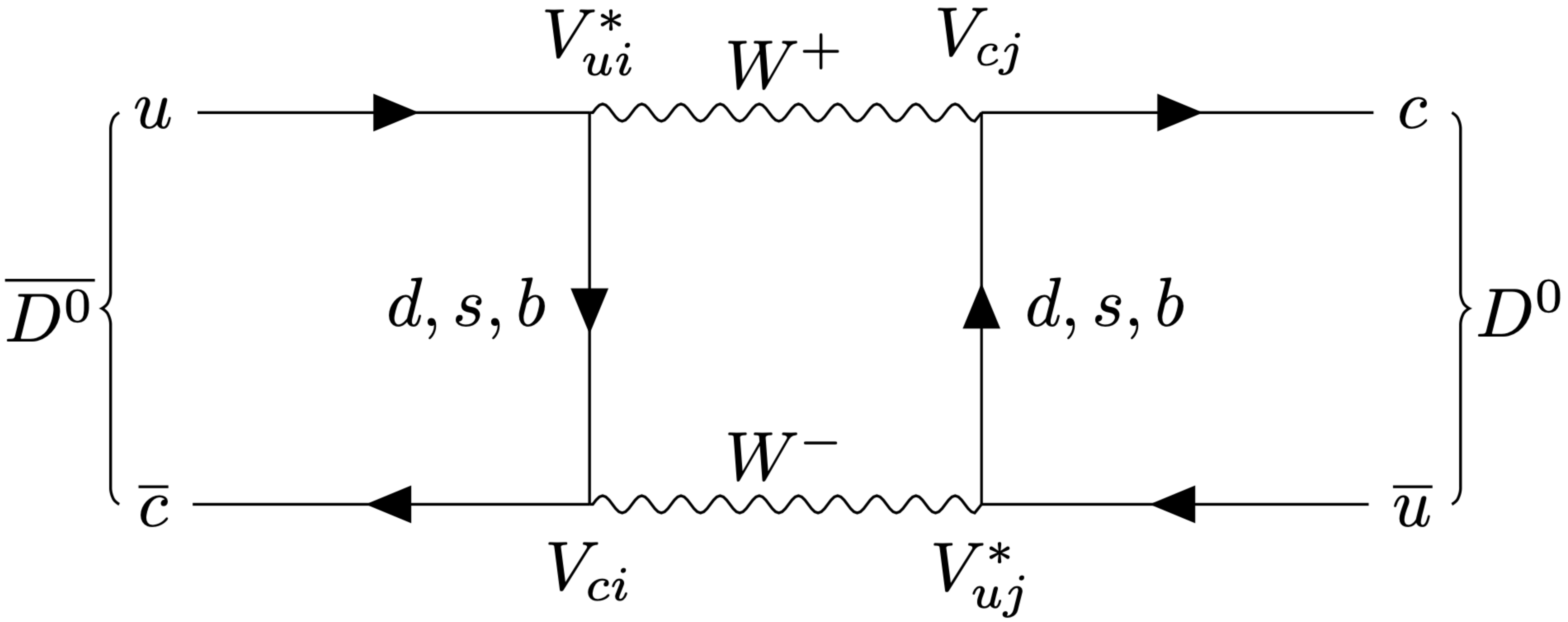
$F_{D[f]}$: CP-even fractions

BEAUTY OBSERVABLES

Approximate Universality

The dispersive and absorptive parts of the antimeson-meson transition amplitude can be decomposed as

$$\Gamma_{12}^{SM} = \sum_{i,j=d,s} \lambda_{uc}^i \lambda_{uc}^j \Gamma_{ij} \qquad M_{12}^{SM} = \sum_{i,j=d,s,b} \lambda_{uc}^i \lambda_{uc}^j M_{ij}$$



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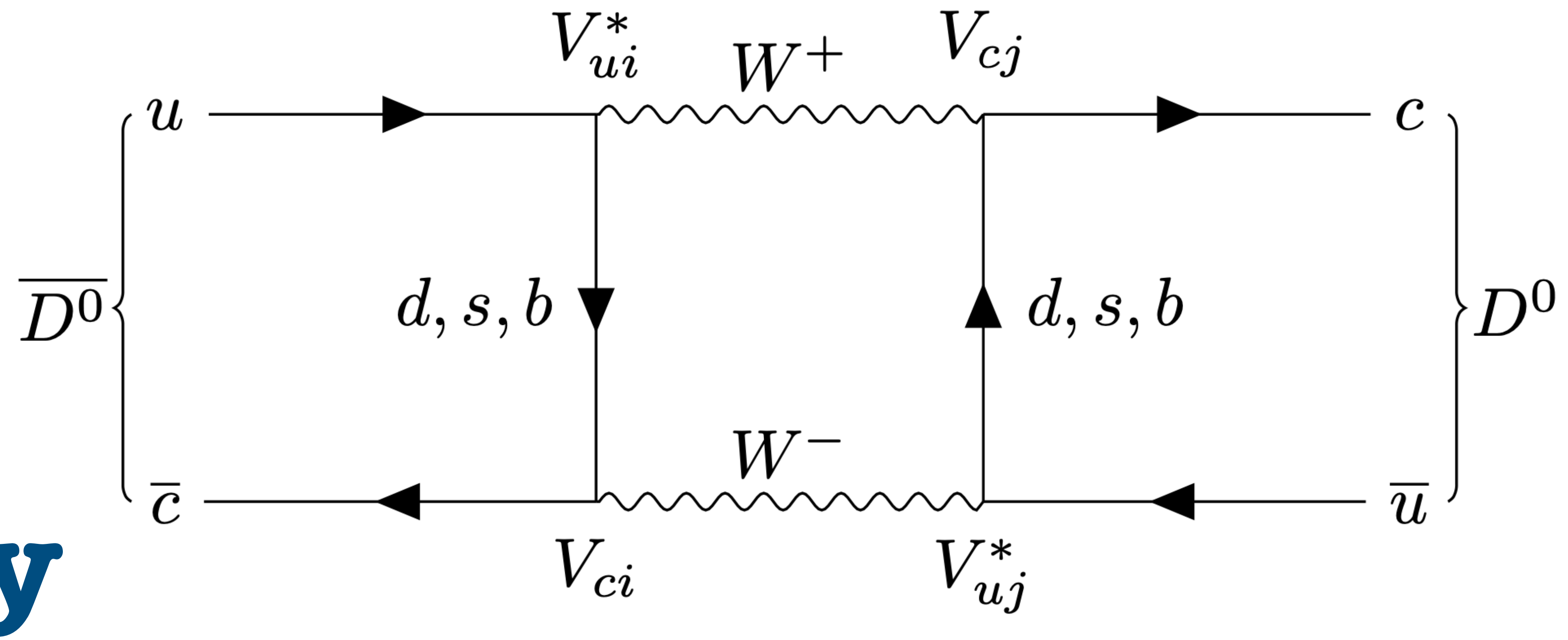

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U-spin decomposition + CKM hierarchy

Dominant

$$\Gamma_{12}^{SM} = \frac{(\lambda^s_{uc} - \lambda^d_{uc})^2}{4}\Gamma_2 + \frac{(\lambda^s_{uc} - \lambda^d_{uc})\lambda^b_{uc}}{2}\Gamma_1 + \frac{\lambda^{b2}_{uc}}{4}\Gamma_0, \quad \Gamma_n = \mathcal{O}(\epsilon^n)$$

$\epsilon \approx 0.3$

U-spin breaking parameter

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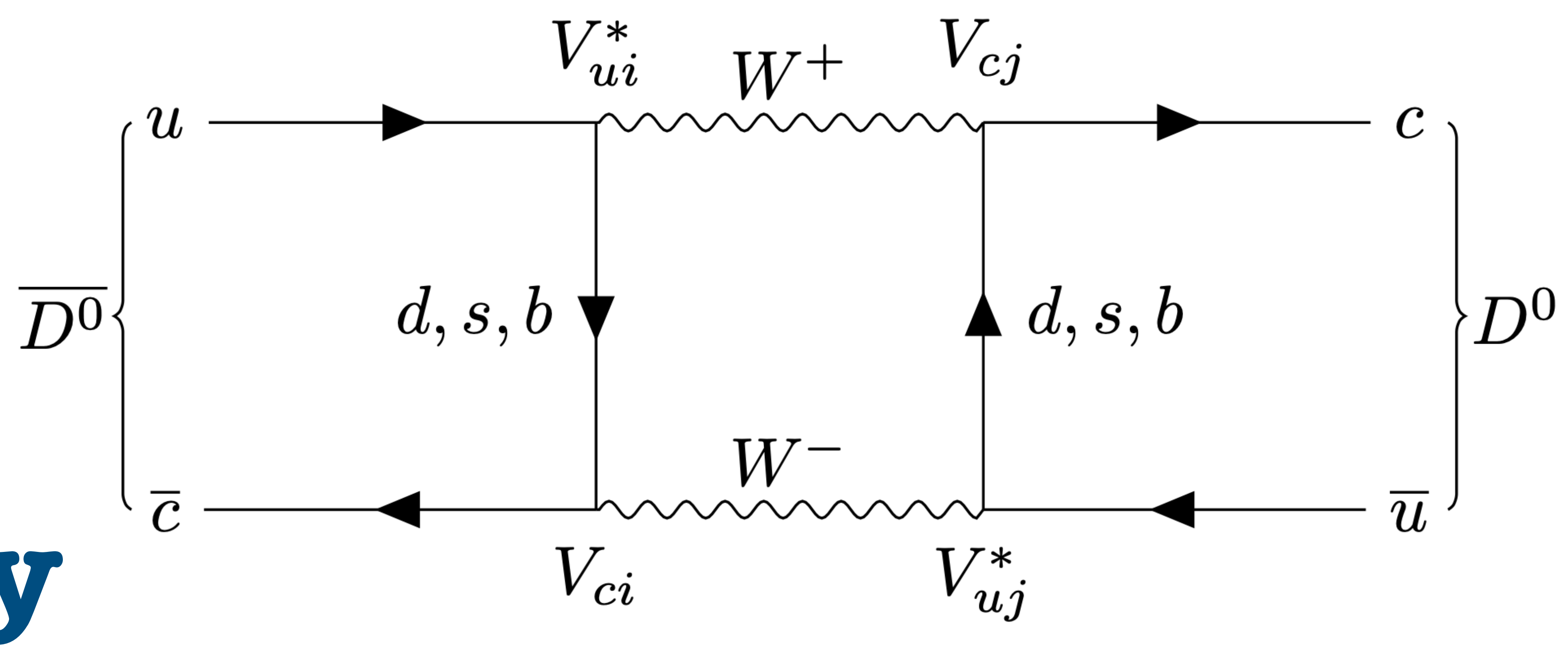

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$\epsilon \approx 0.3$

U-spin breaking parameter

Universal weak phases

We can define two CP violating weak phases with respect the dominant U-spin ($\Delta U = 2$) term

Good approximation for every final states

$\phi_f^{M,\Gamma} \simeq \phi_2^{M,\Gamma}$

$$\phi_2^X = \arg\left[\frac{X_{12}}{X_2(\lambda_{uc}^s - \lambda_{uc}^d)^2/4}\right]\Bigg|_{X=M,\Gamma}$$

SM estimates

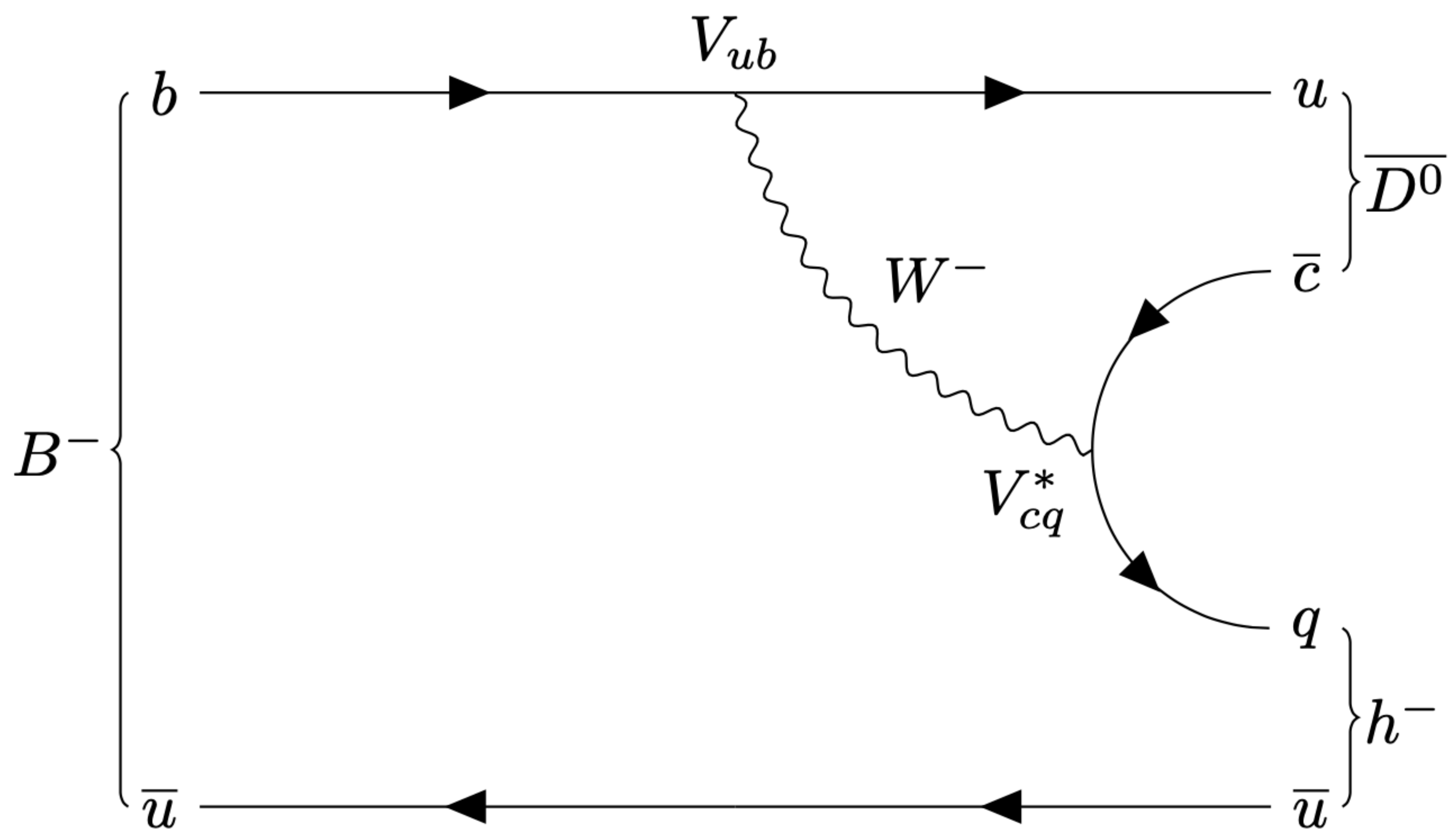
$\phi_2^{M,\Gamma} \approx 0.13^\circ$

B meson cascade decays

We provided additional information about the decay parameters of the D mesons by considering also processes involving the beauty quark, as already shown by LHCb (LHCb-CONF-2022-002)

B CASCADE DECAYS

$$B \to [f]_D h$$



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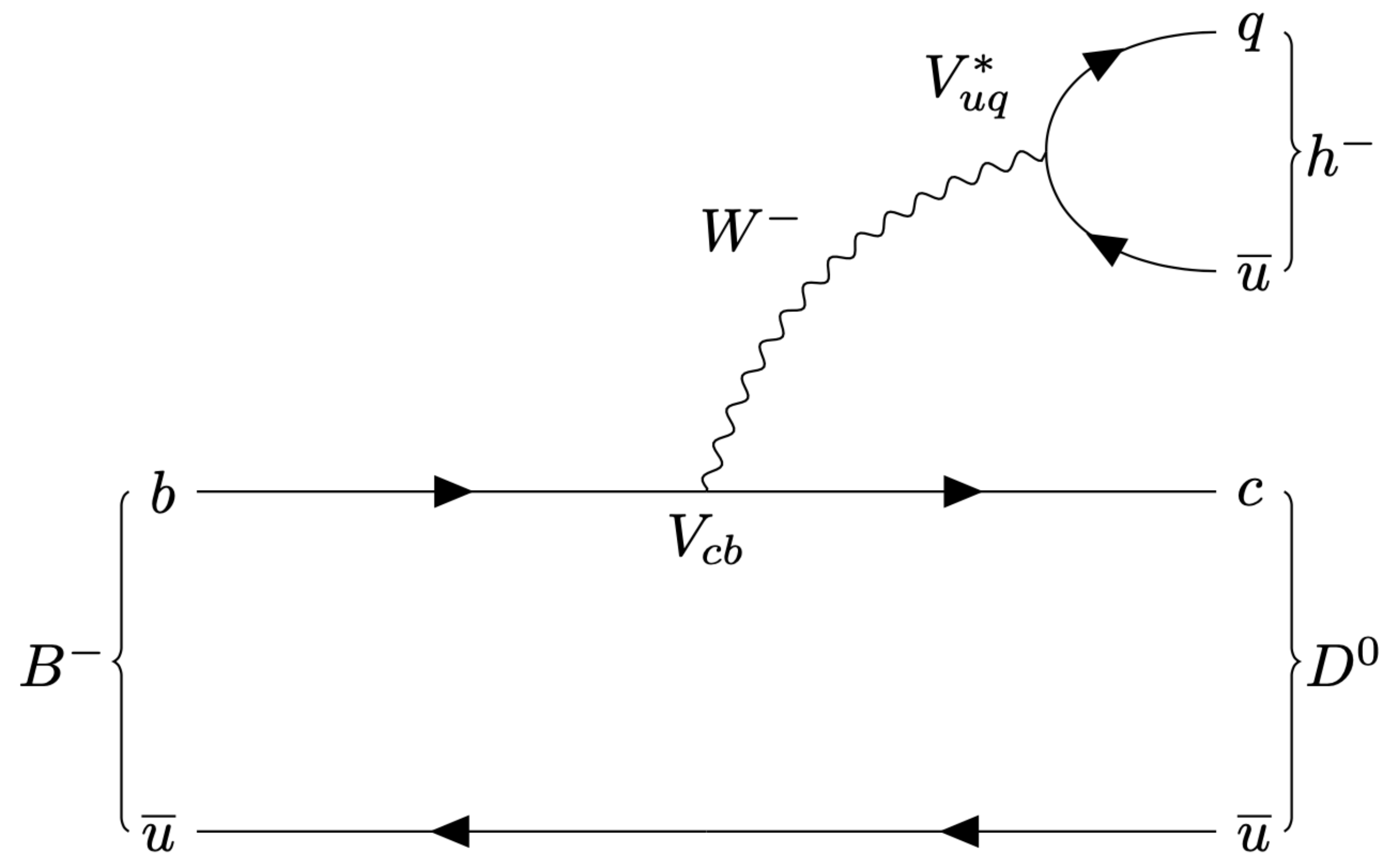

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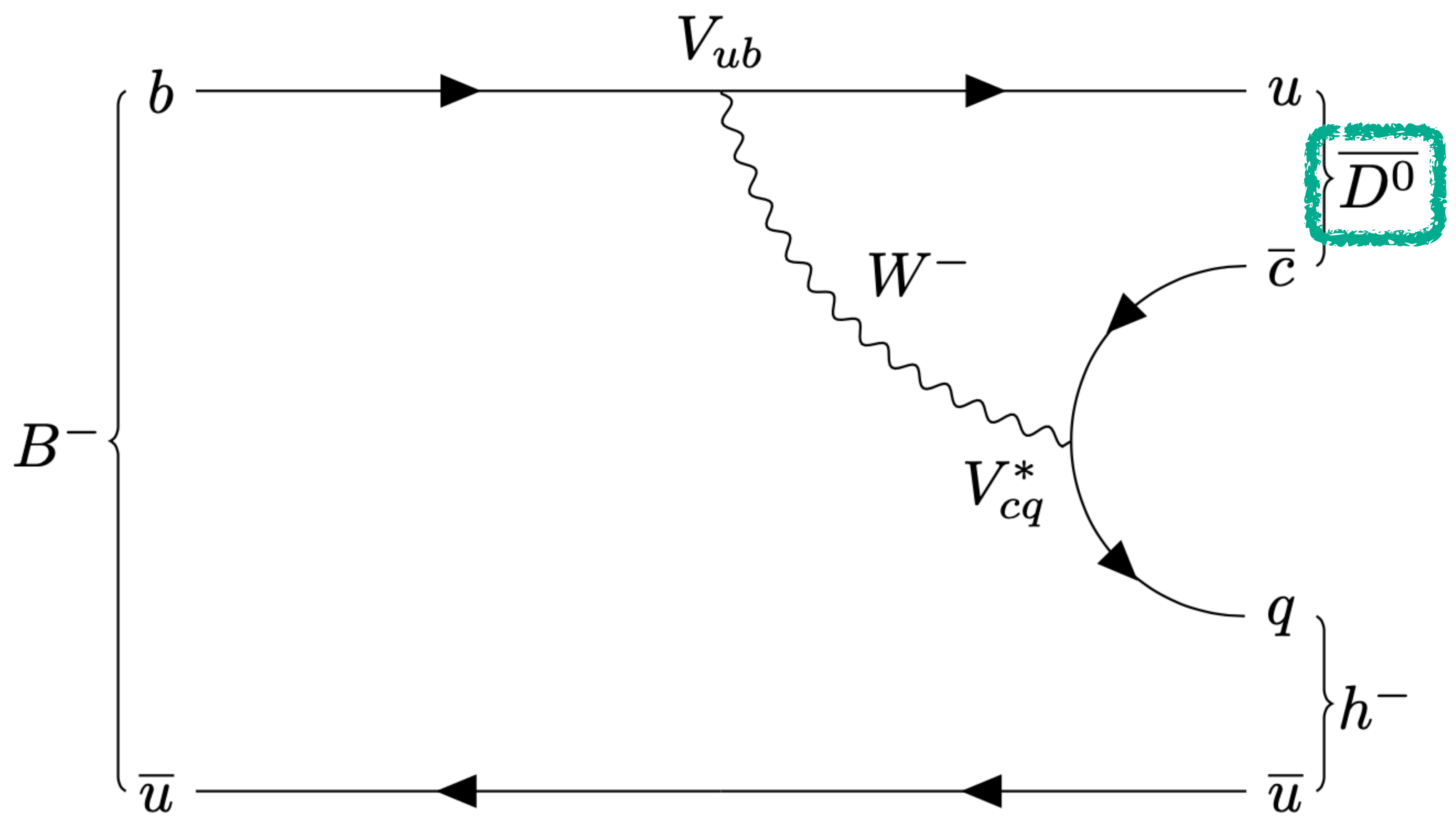

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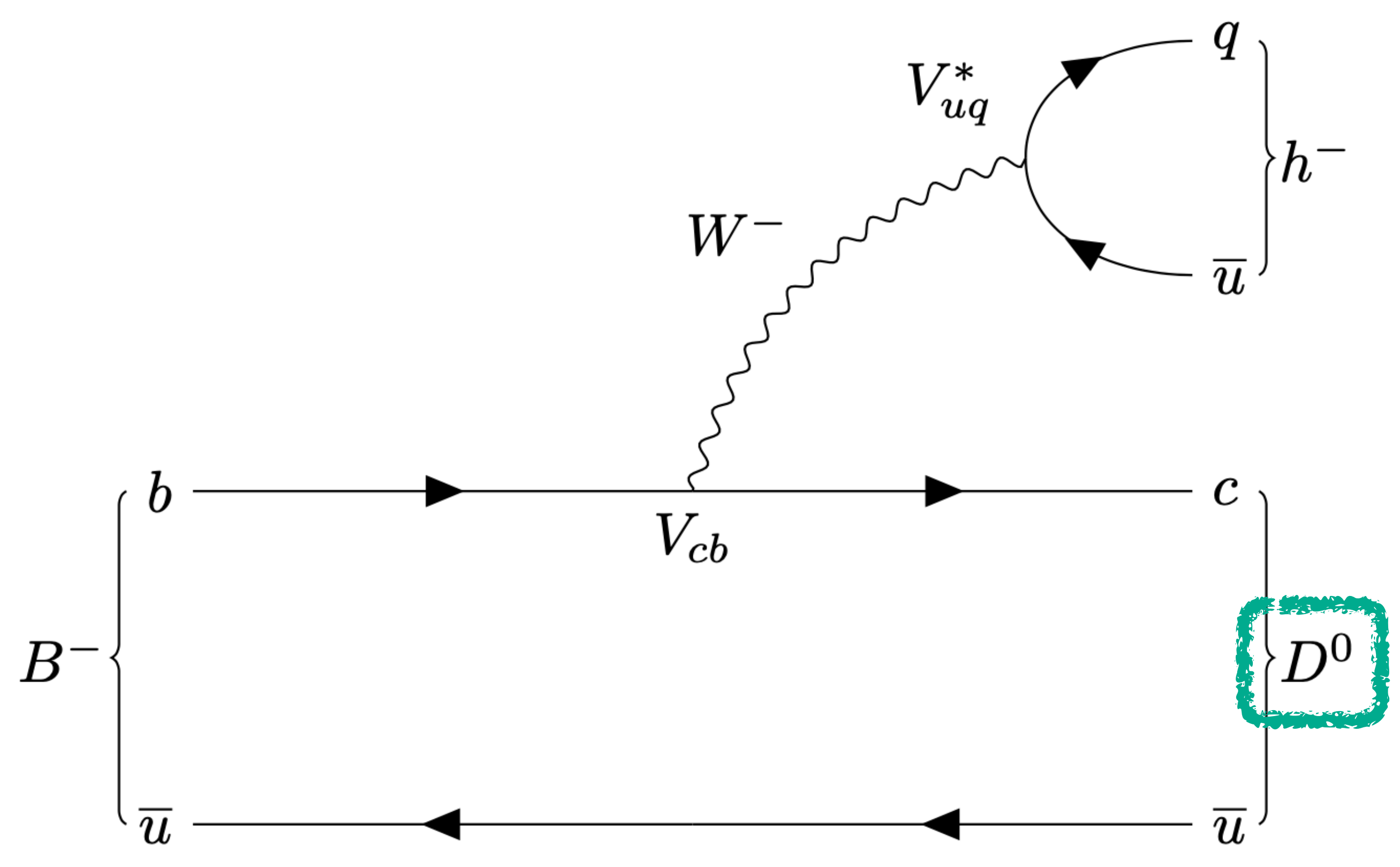

$D \to f$ decays

Parametrizing the amplitudes in terms of

Ratio of the magnitudes: $r_{D[f]}$

Strong phase: Δ_f

Mixing parameters: x_{12}, y_{12}

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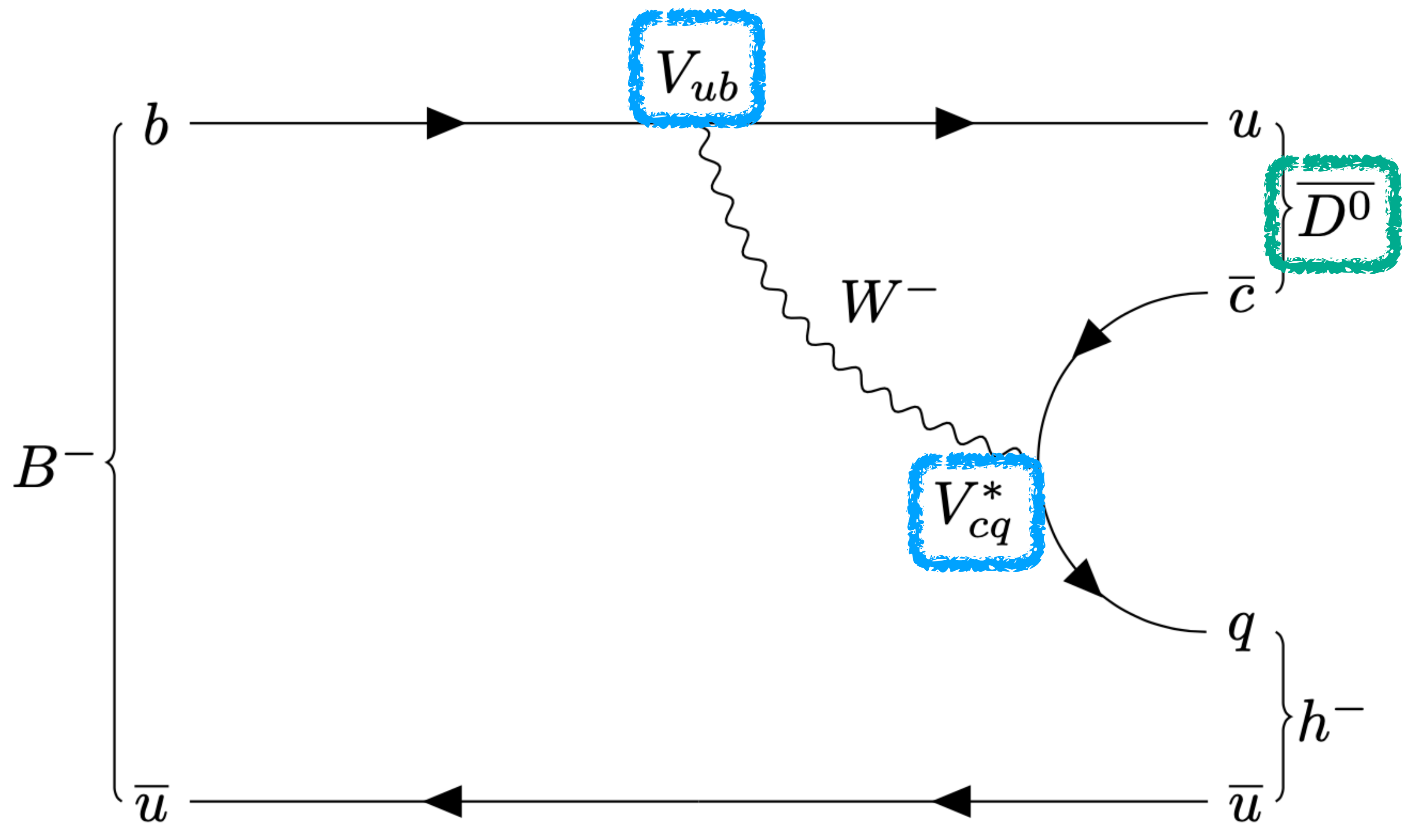

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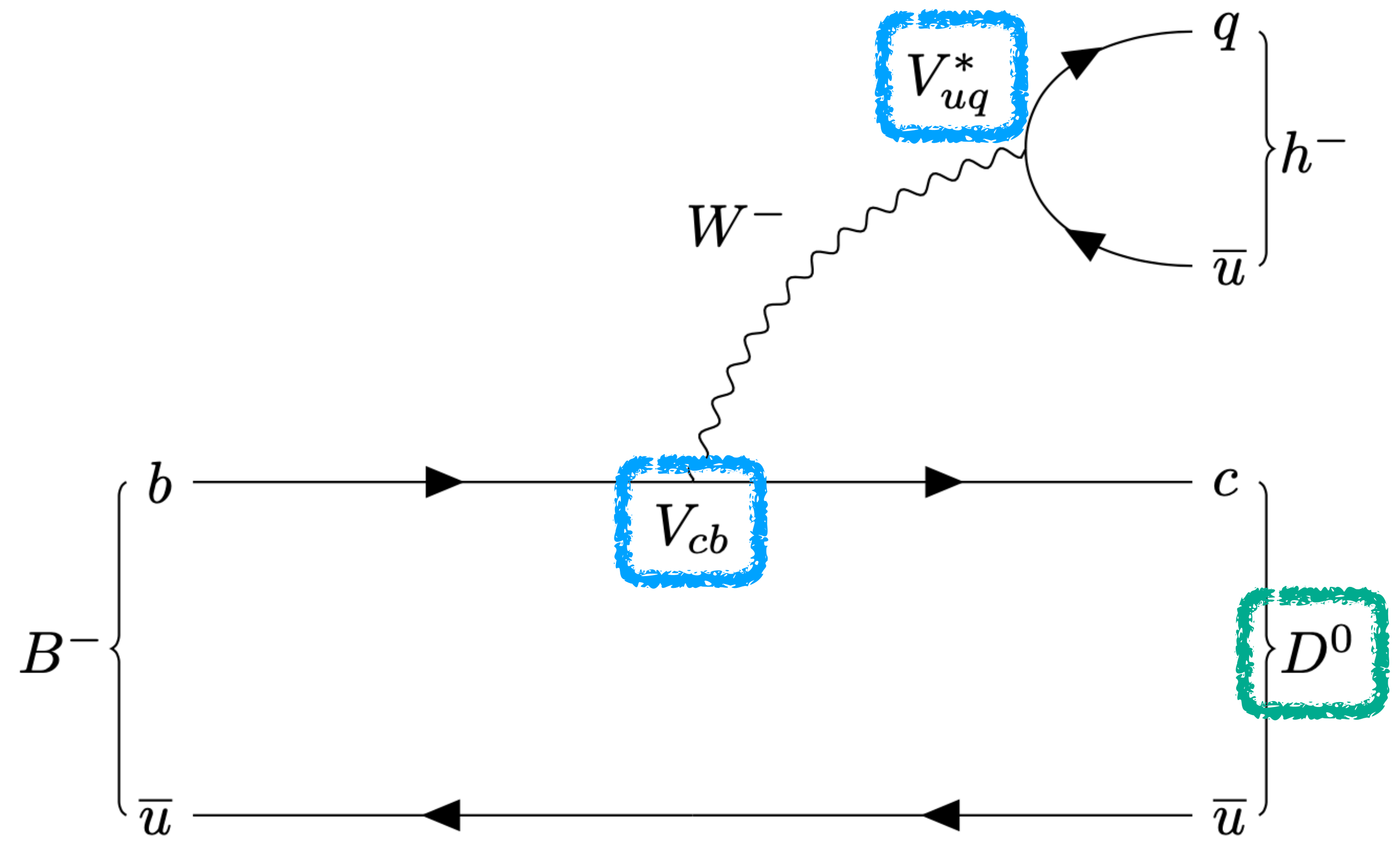

$B \to D$ decays: sensitivity to γ

Parametrizing the amplitudes in terms of

Ratio of the magnitudes: $r_{B[Dh]}$

Strong phase: $\delta_{B[Dh]}$

Weak phase: $\arg[V_{ub}V_{uq}V^*_{cb}V^*_{cq}] \approx -\gamma$

$D \to f$ decays

Parametrizing the amplitudes in terms of

Ratio of the magnitudes: $r_{D[f]}$

Strong phase: Δ_f

Mixing parameters: $x_{12},\ y_{12}$

GLW/ADS Observables

SENSITIVITY TO γ

(quasi-) GLW

$$B \to D^0 h$$
$$B \to \overline{D^0} h$$

(quasi-) ADS

$$B \to D^0 h$$
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GLW/ADS Observables

(quasi-) GLW

SENSITIVITY TO γ

SENSITIVITY TO CHARM DECAY PARAMETERS

(quasi-) ADS

$B \to D^0 h \to [f_{CP}]_D h$

$B \to \overline{D^0} h \to [f_{CP}]_D h$

$$f_{CP} = \pi^+\pi^-(\pi^0),\ K^+K^-(\pi^0),\ \pi^+\pi^-\pi^+\pi^-,\ K^+K^-\pi^+\pi^-$$

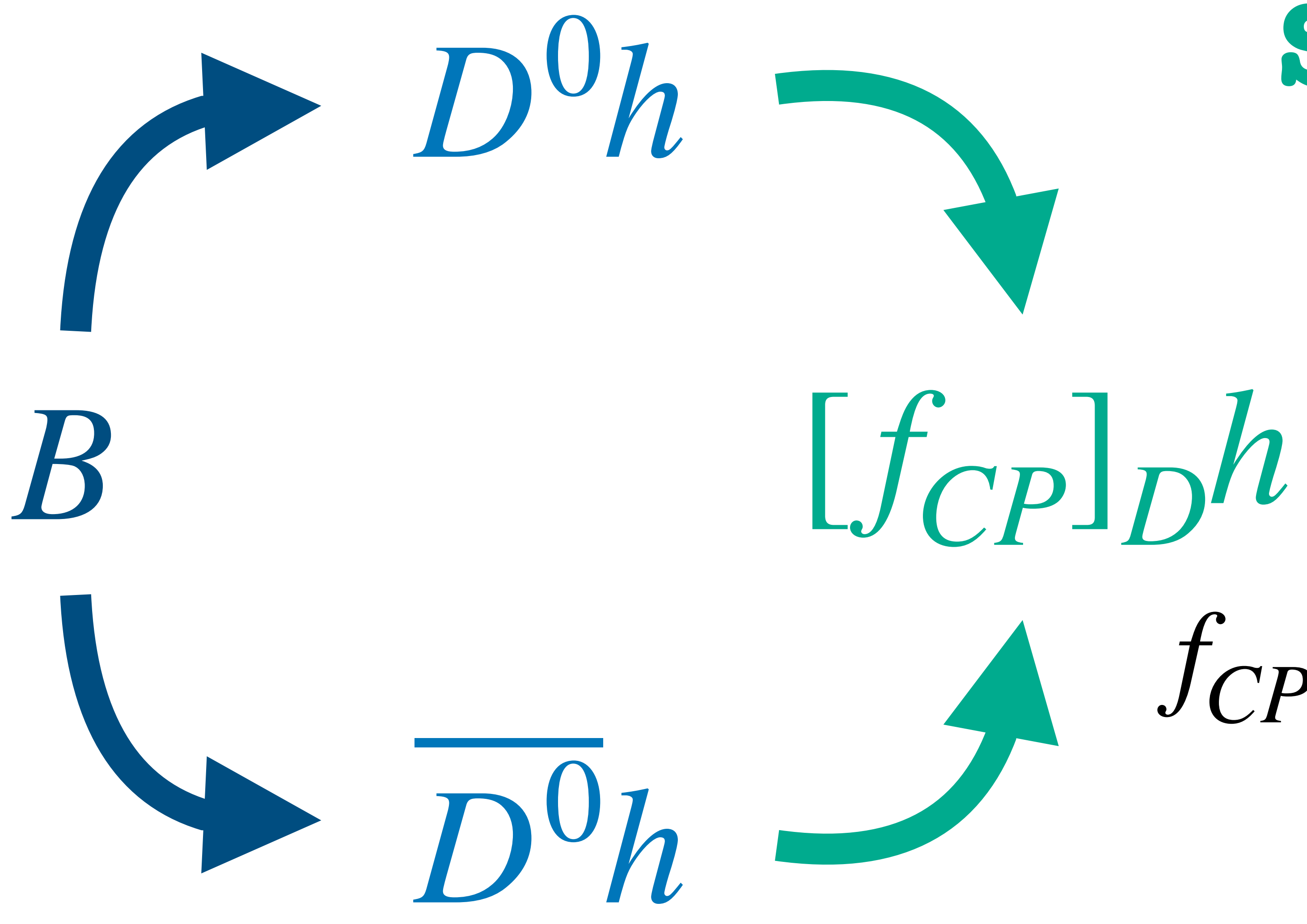
$B \to D^0 h \to [f]_D h$

$B \to \overline{D^0} h \to [f]_D h$

$$f = K^-\pi^+(\pi^0),\ K^-\pi^+\pi^+\pi^-$$

GLW/ADS Observables

(quasi-) GLW



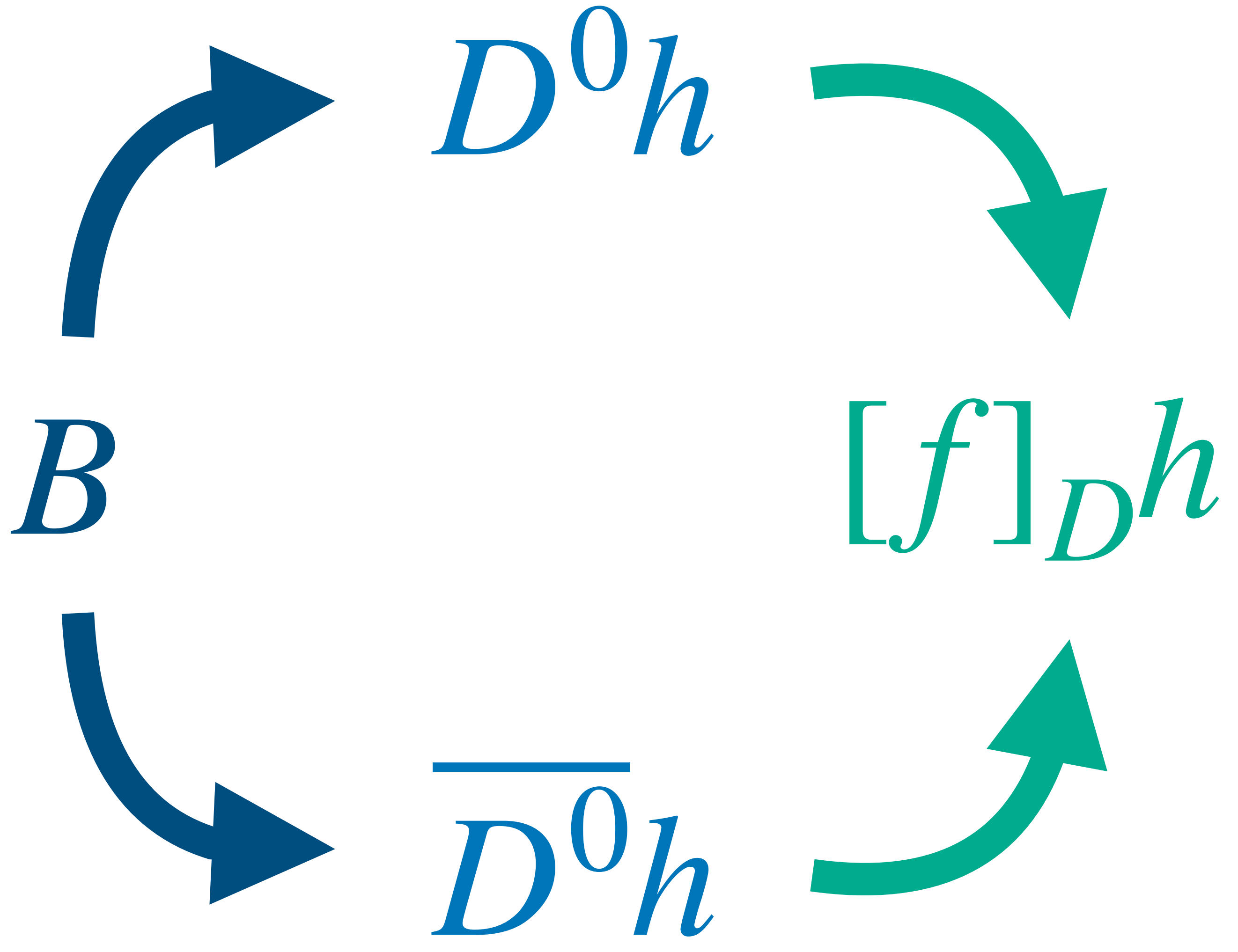
SENSITIVITY TO γ

SENSITIVITY TO CHARM DECAY PARAMETERS

$f = K^-\pi^+(\pi^0),\ K^-\pi^+\pi^+\pi^-$

$f_{CP} = \pi^+\pi^-(\pi^0),\ K^+K^-(\pi^0),$
$\pi^+\pi^-\pi^+\pi^-,\ K^+K^-\pi^+\pi^-$

(quasi-) ADS



Decay rates to first order in mixing and CPV parameters

$$\Gamma(B \to [f_{CP}]_D h) \propto 1 + r^2_{B[Dh]} + \text{Mixing part} + 2\kappa_{B[Dh]} r_{B[Dh]} (2F_{D[f_{CP}]} - 1)\cos(\delta_{B[Dh]} - \gamma)$$

$$\Gamma(B \to [f]_D h) \propto 1 + r^2_{B[Dh]} r^2_{D[f]} + \text{Mixing part} + 2\kappa_{D[f]} \kappa_{B[Dh]} r_{B[Dh]} r_{D[f]} \cos(\Delta_f + \delta_{B[Dh]} - \gamma)$$

Observables!!
Ratios of decay rates
(e.g. CP Asymmetries)

GGSZ Observables

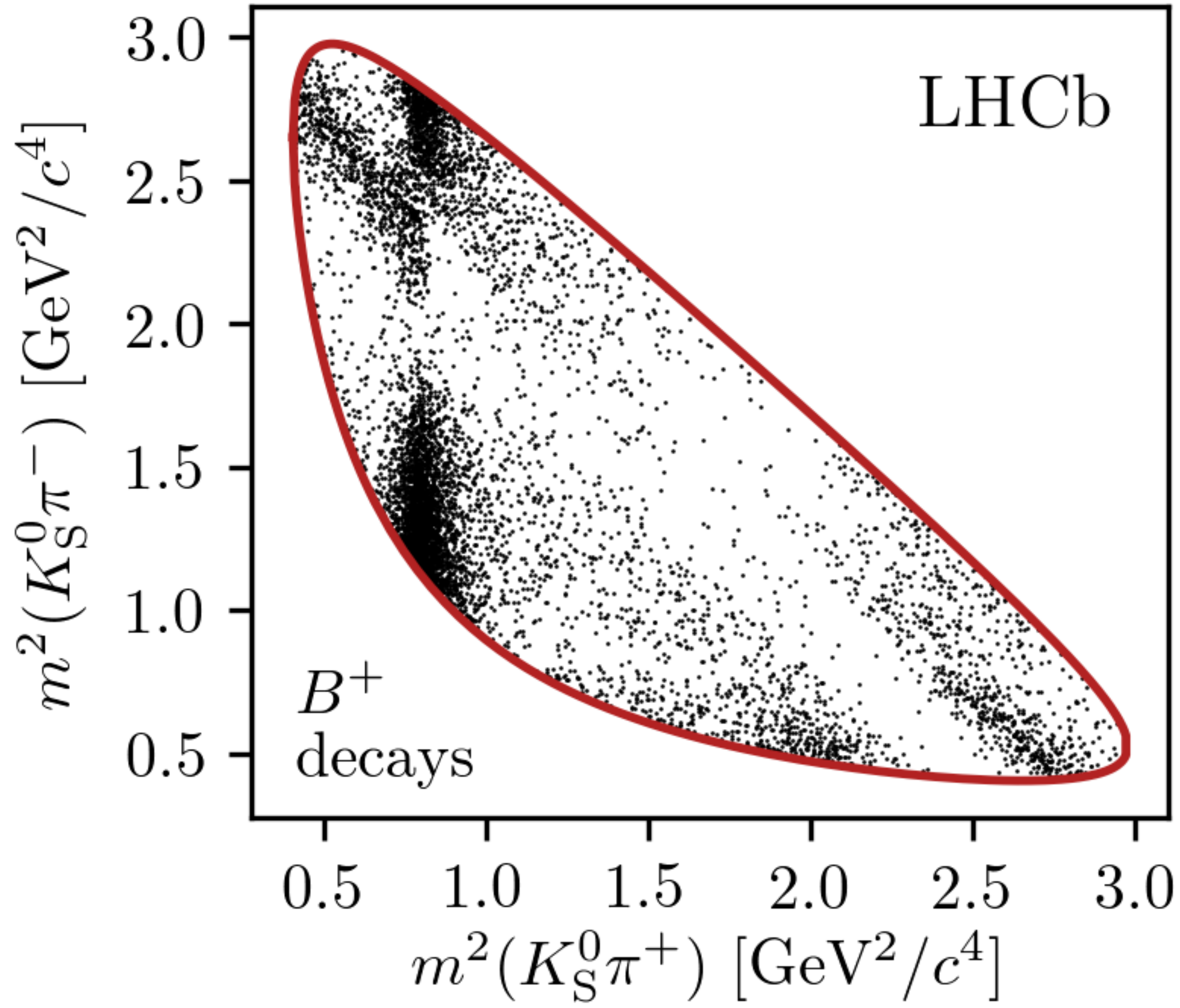
It is possible to study also B cascade decays, with a three-body final state of the D meson (e.g. $f = K_s^0 \pi^+ \pi^-$).

Measuring the decay rate at a phase space point

$$d\Gamma\left(\overset{(-)}{B} \to [f]_D \overset{(-)}{h}\right)/dp$$

Dalitz plot

from 2010.08483



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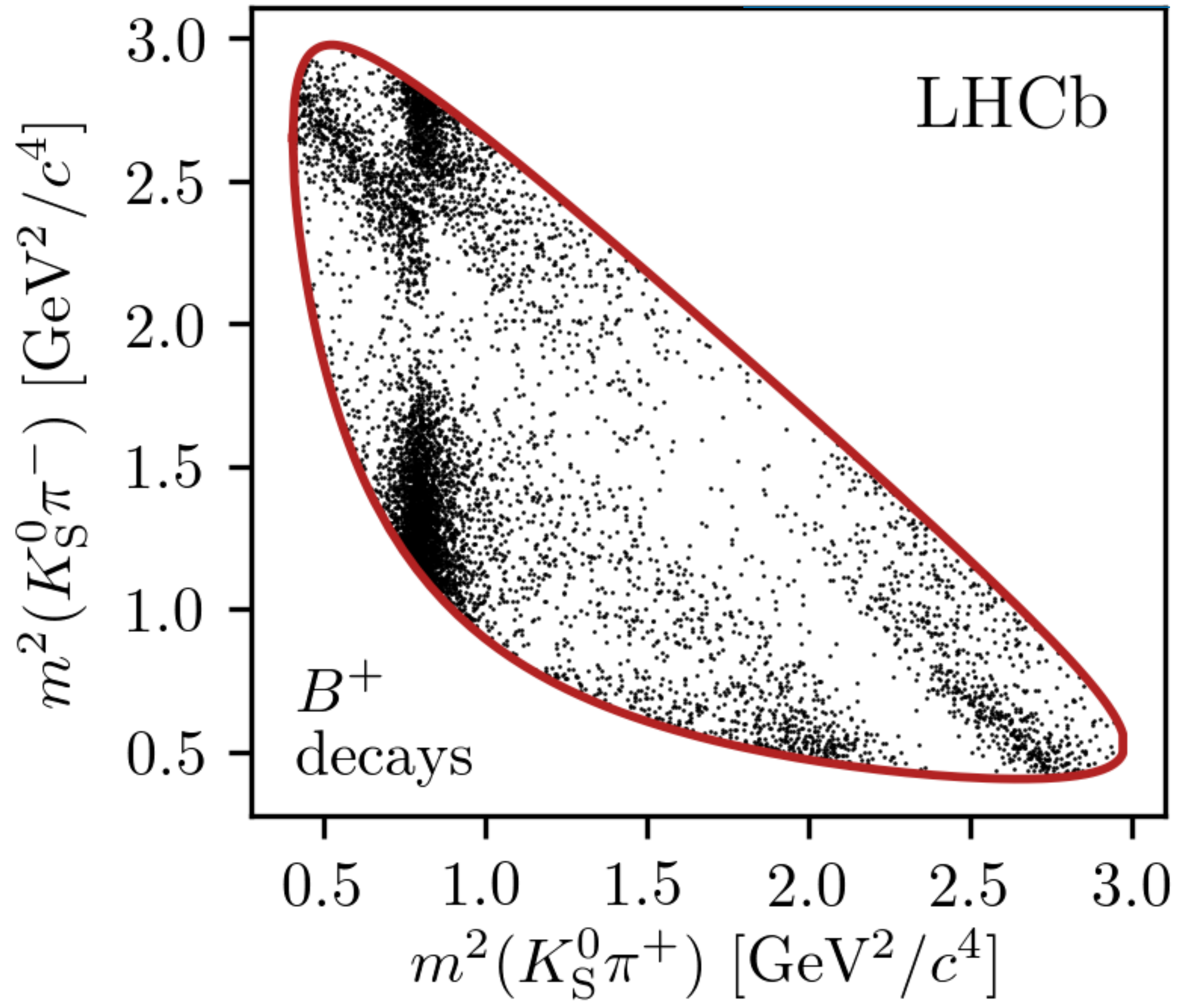

Model dependent approach: The decay rate is fitted using some model for the D decay amplitudes.

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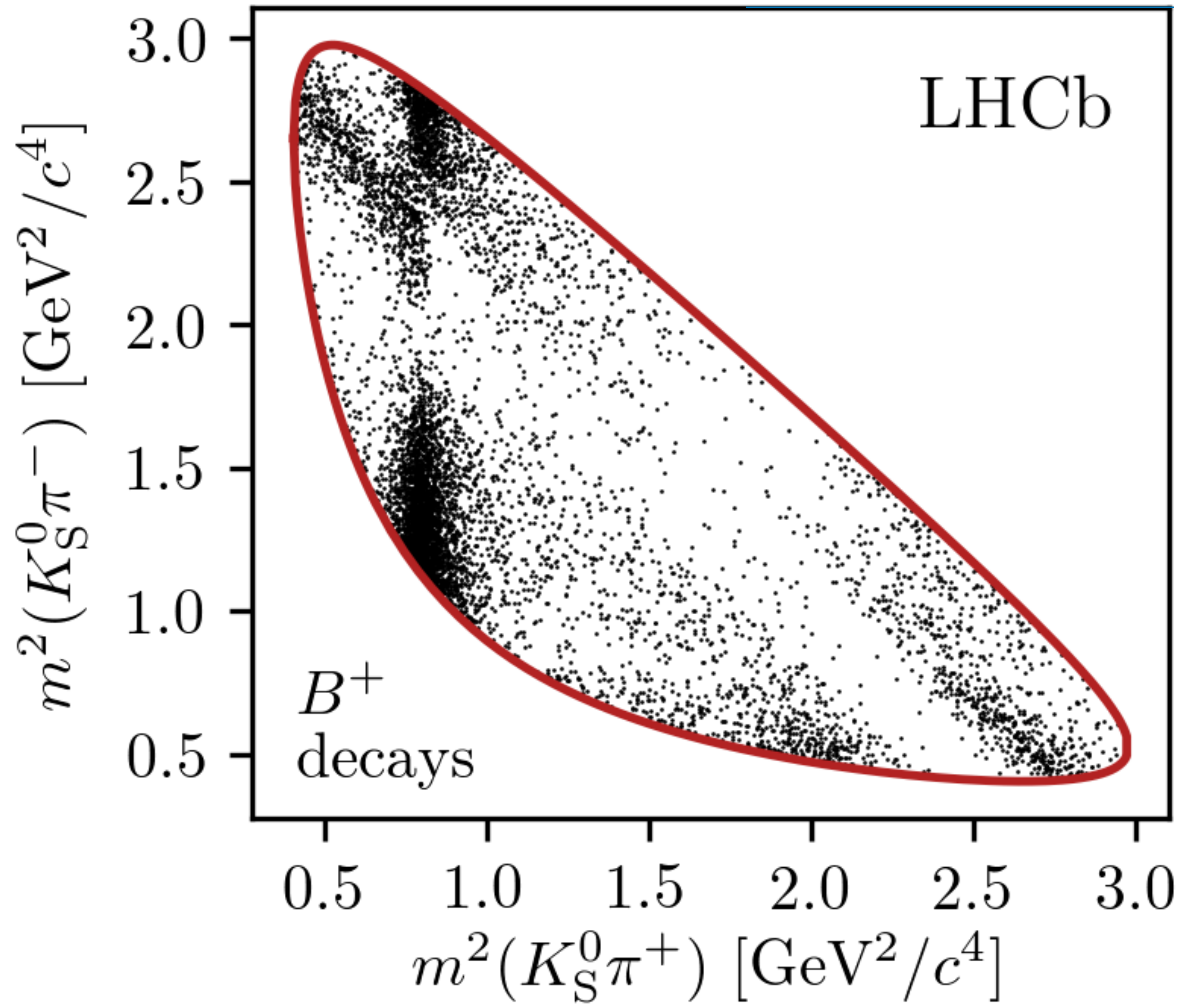


Model dependent approach: The decay rate is fitted using some model for the D decay amplitudes.

Model independent approach: Integrating over the bins and solving a system of $4k$ equations $\Gamma_{\pm i}(\overset{(-)}{B} \to [f]_D \overset{(-)}{h})$ for $2k + 4$ unknowns.

Observables!!

$$x_{\pm}^{Dh} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm}^{Dh} = r_B \sin(\delta_B \pm \gamma)$$

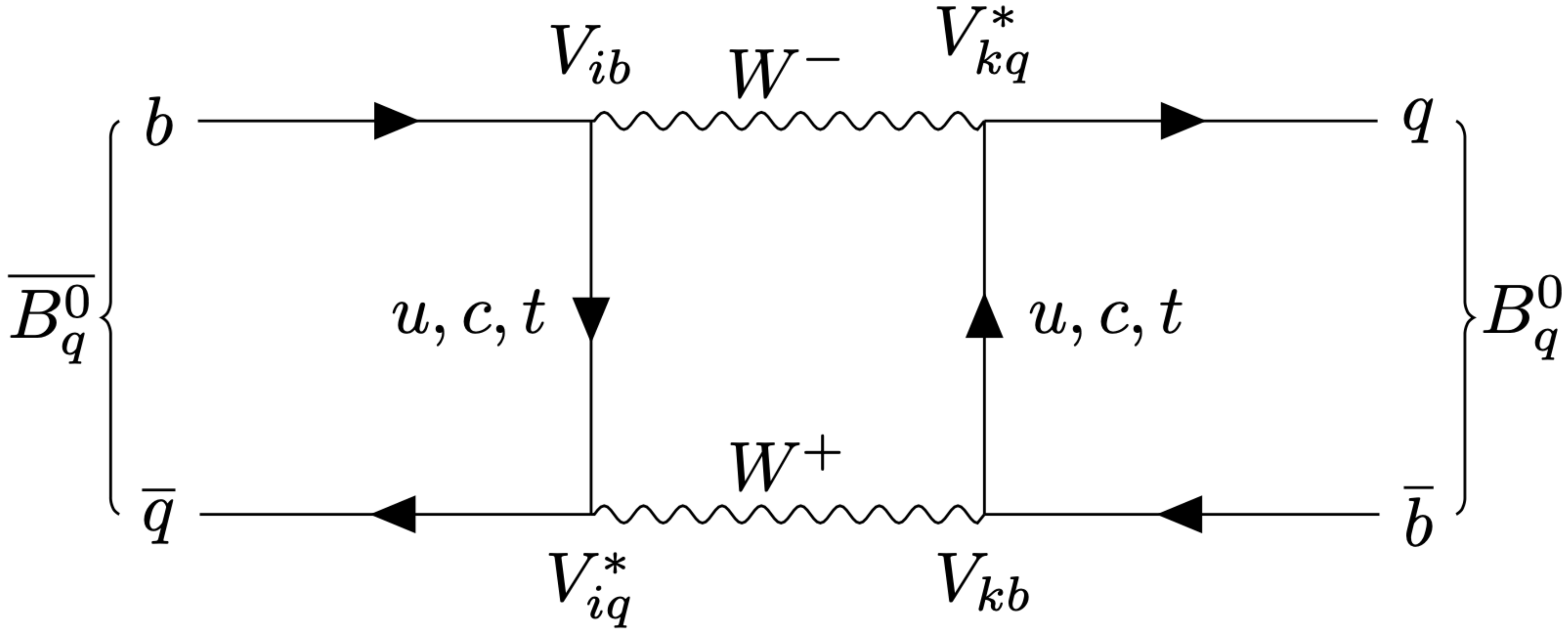
Neutral B mesons

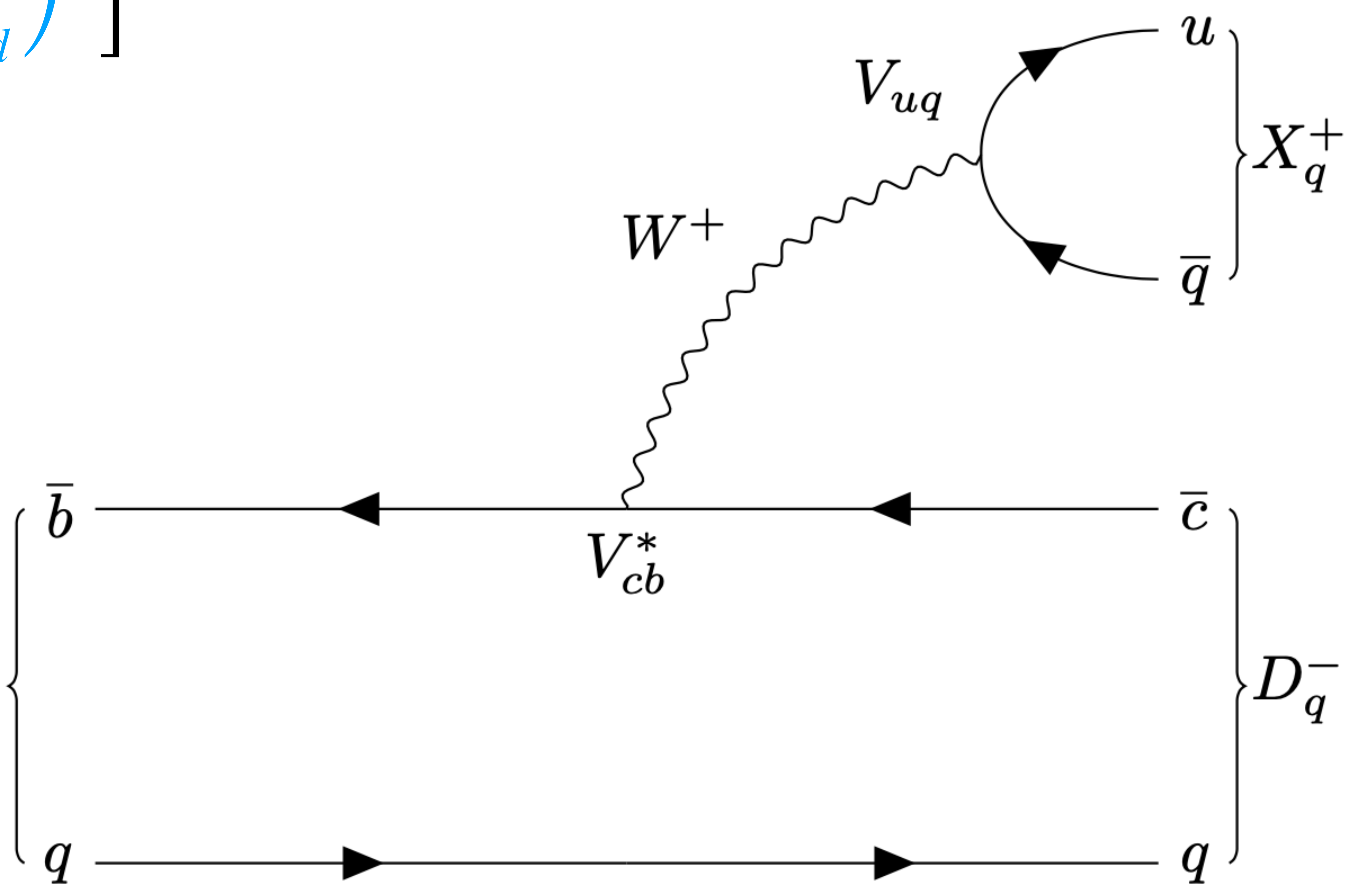
Additional information on the CKM angle γ comes from neutral B decays to charmed mesons

Interference between mixing and decay: $B_q^0 \to D_q^{\pm} X_q^{\mp}, \quad q = d, s, \quad X_q = \pi,\ K,\ K\pi\pi$

$$\phi_\lambda = \arg\left[\left(-\frac{V_{tb}^* V_{tq}}{V_{cb}^* V_{cq}}\right)^2 \left(-\frac{V_{ub} V_{ud}^*}{V_{cb} V_{cd}^*}\right) \left(-\frac{V_{cq} V_{uq}^*}{V_{cd} V_{ud}^*}\right)\right]$$

$q = d:$	-2β	$-\gamma$	π
$q = s:$	$2\beta_s$	$-\gamma$	$\mathcal{O}(\lambda^4)$





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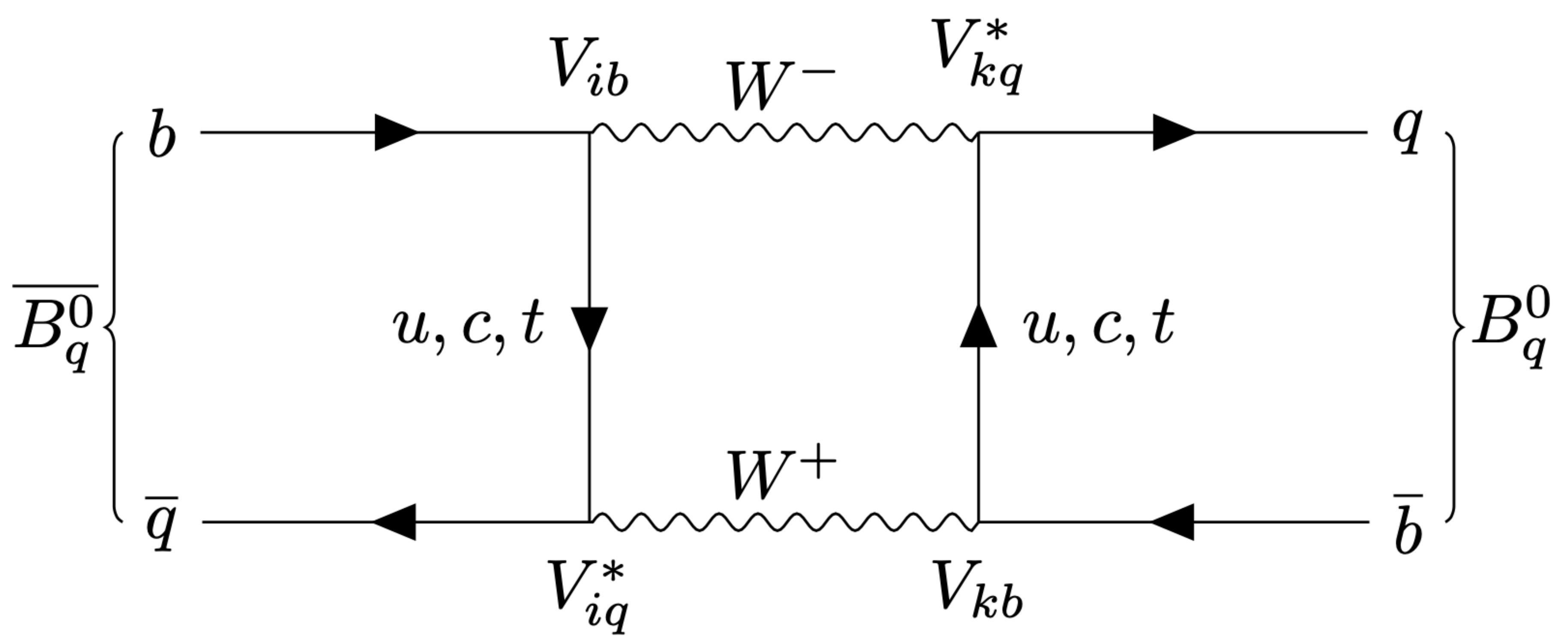
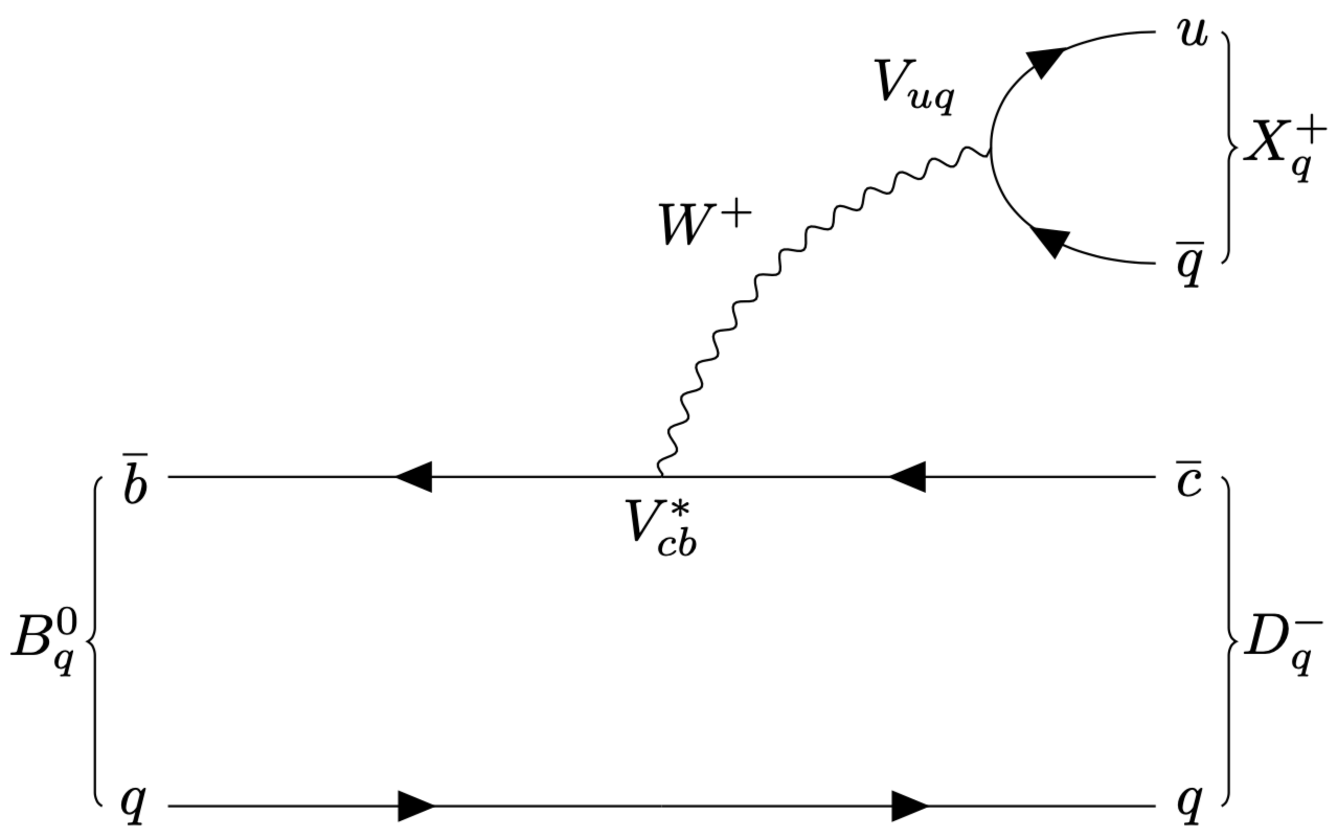
$q = d:$ $\quad -2\beta \quad -\gamma \quad \pi$

$q = s:$ $\quad 2\beta_s \quad -\gamma \quad \mathcal{O}(\lambda^4)$



$$\frac{\mathrm{d}\Gamma(\overset{(-)}{B_q^0} \to f)}{\mathrm{d}t} \propto \cosh(y\Gamma t) - G_f \sinh(y\Gamma t) \pm C_f \cos(x\Gamma t) \mp S_f \sin(x\Gamma t)$$



Observables!! $\quad G_f \propto \cos(\Delta_{B_q^0} + \phi_\lambda)$

$S_f \propto \sin(\Delta_{B_q^0} + \phi_\lambda) \quad C_f$

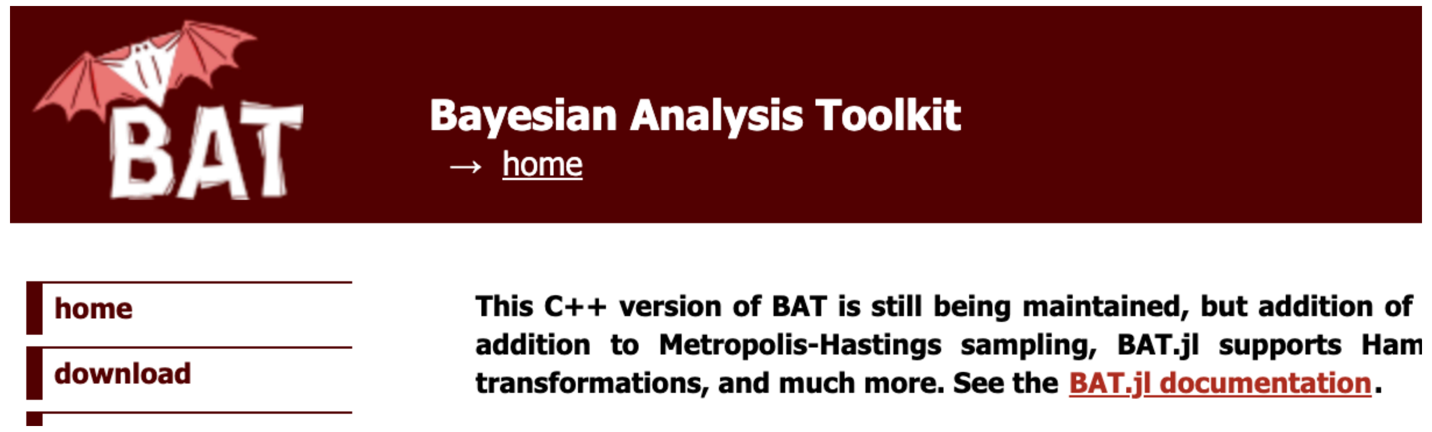
Statistical treatment

We combine all the observables in a **Bayesian framework** to determine the posterior pdf and the marginalized distributions.

$$P(\overrightarrow{\lambda}\,|\,\mathbf{O})$$

POSTERIOR PDF

We overloaded the classes present in the BAT library, sampling configurations of the parameters from the posterior through a Metropolis algorithm.



https://bat.mpp.mpg.de/

Statistical treatment

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GAUSSIAN LIKELIHOOD

$$P(\mathbf{O}\,|\,\vec{\lambda}) \propto \prod_{j=1}^{N_{exp}} \exp\left[-\left(O_i^{(j)}(\vec{\lambda}) - O_i^{(j)}\right)^T (V^{(j)})^{-1}_{ik} \left(O_k^{(j)}(\vec{\lambda}) - O_k^{(j)}\right)\right]$$

$$P(\vec{\lambda}\,|\,\mathbf{O}) \propto P(\mathbf{O}\,|\,\vec{\lambda})$$

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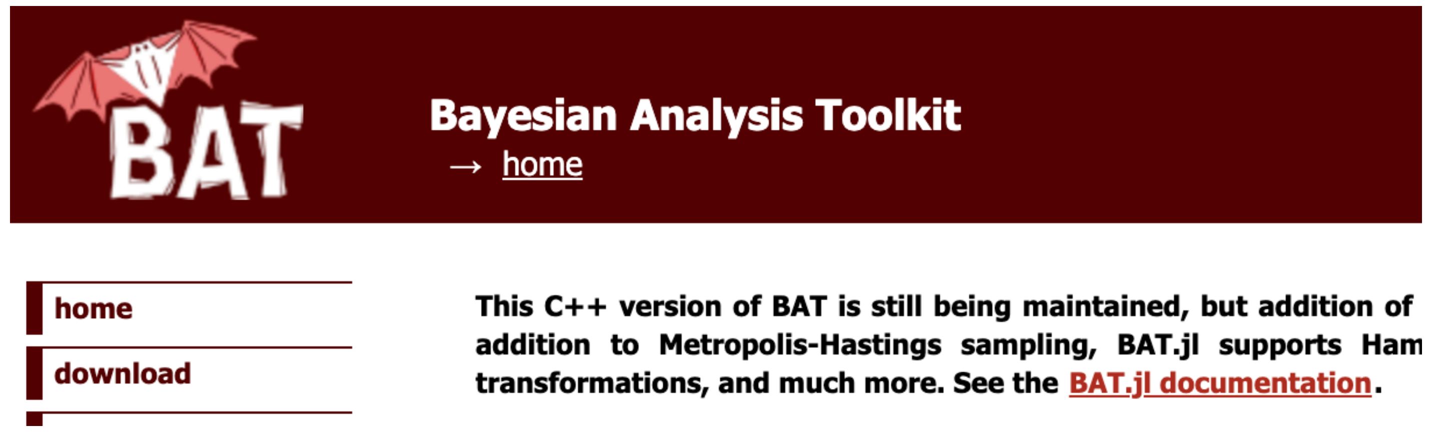

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$$P(\vec{\lambda}\,|\,\mathbf{O}) \propto P(\mathbf{O}\,|\,\vec{\lambda})\; P_0(\vec{\lambda})$$

FLAT PRIOR

We choose uniform priors, according to the physical ranges of the parameters

$$P_0(\vec{\lambda}) = const$$

POSTERIOR PDF

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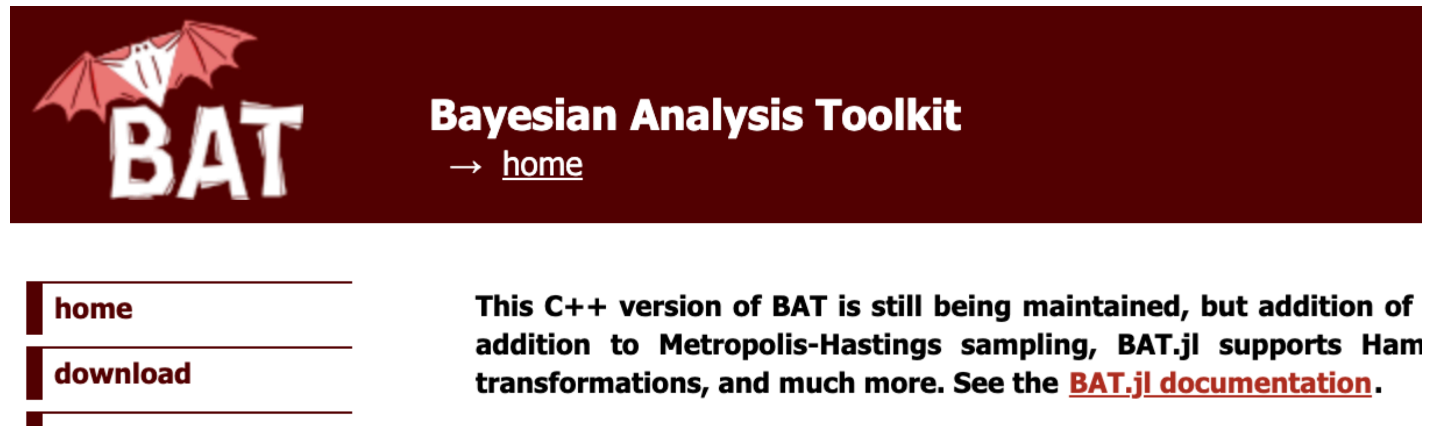
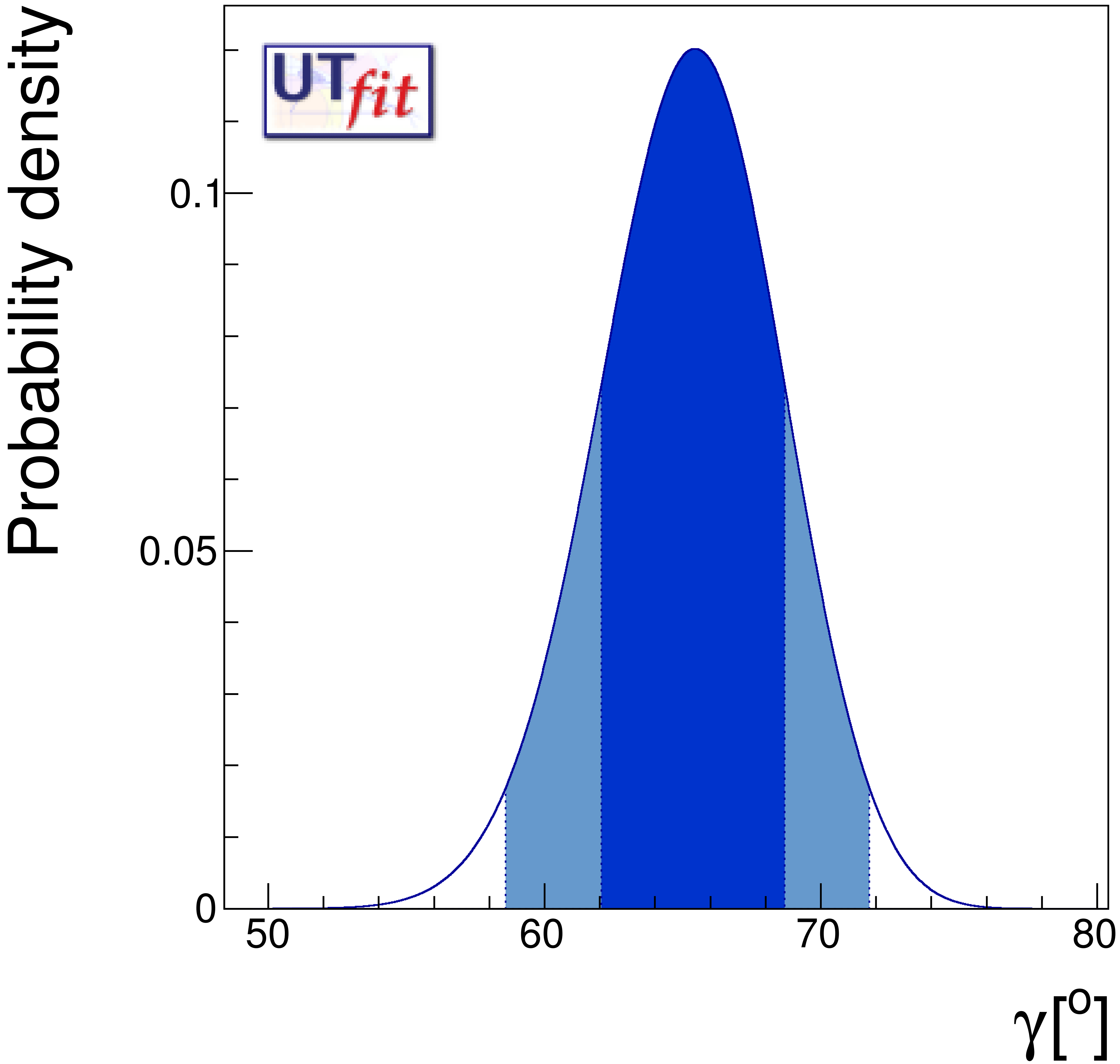


https://bat.mpp.mpg.de/

CKM angle γ using all the inputs

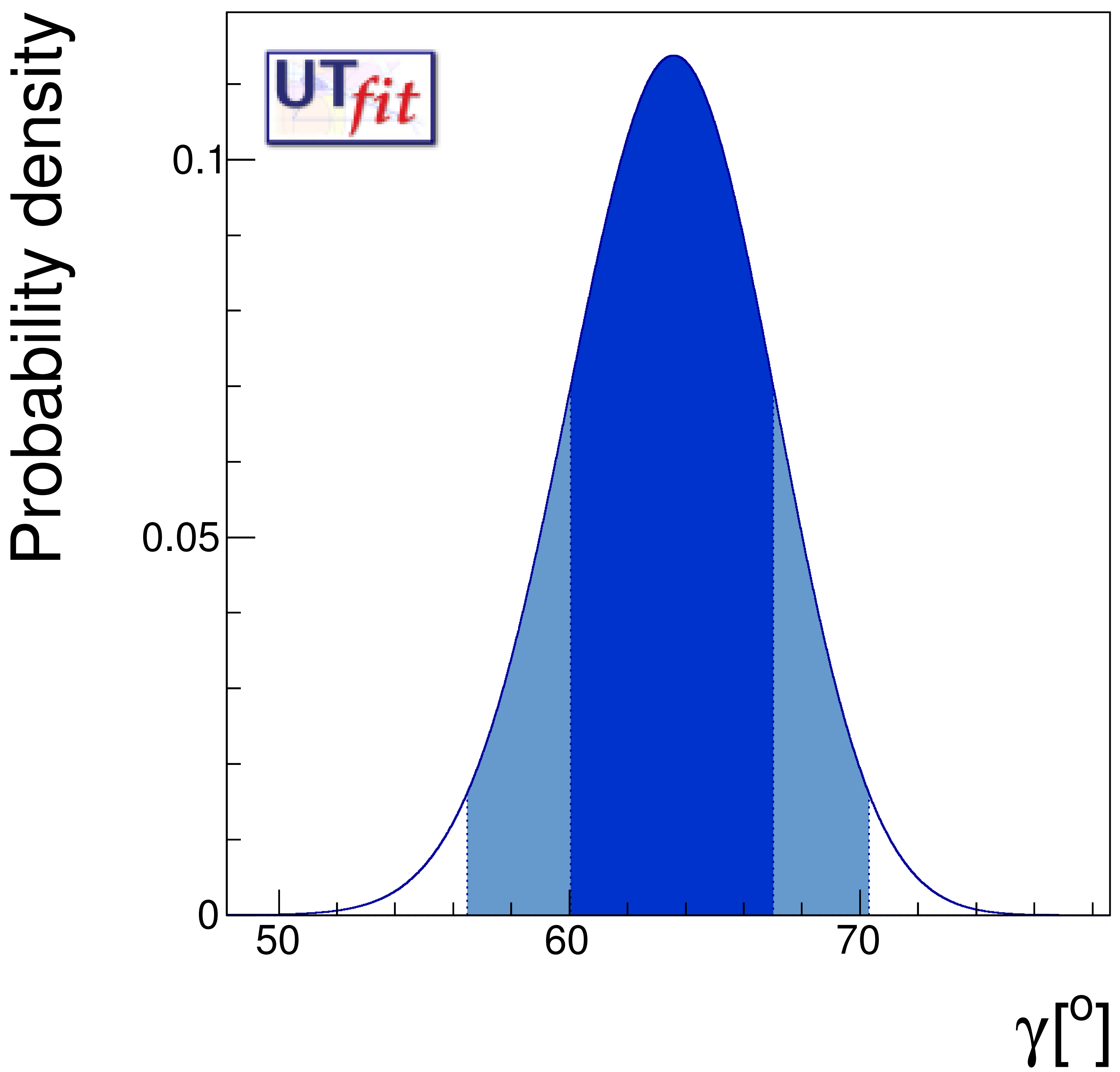


FIT RESULT

$$\gamma = (65.4 \pm 3.3)^\circ$$

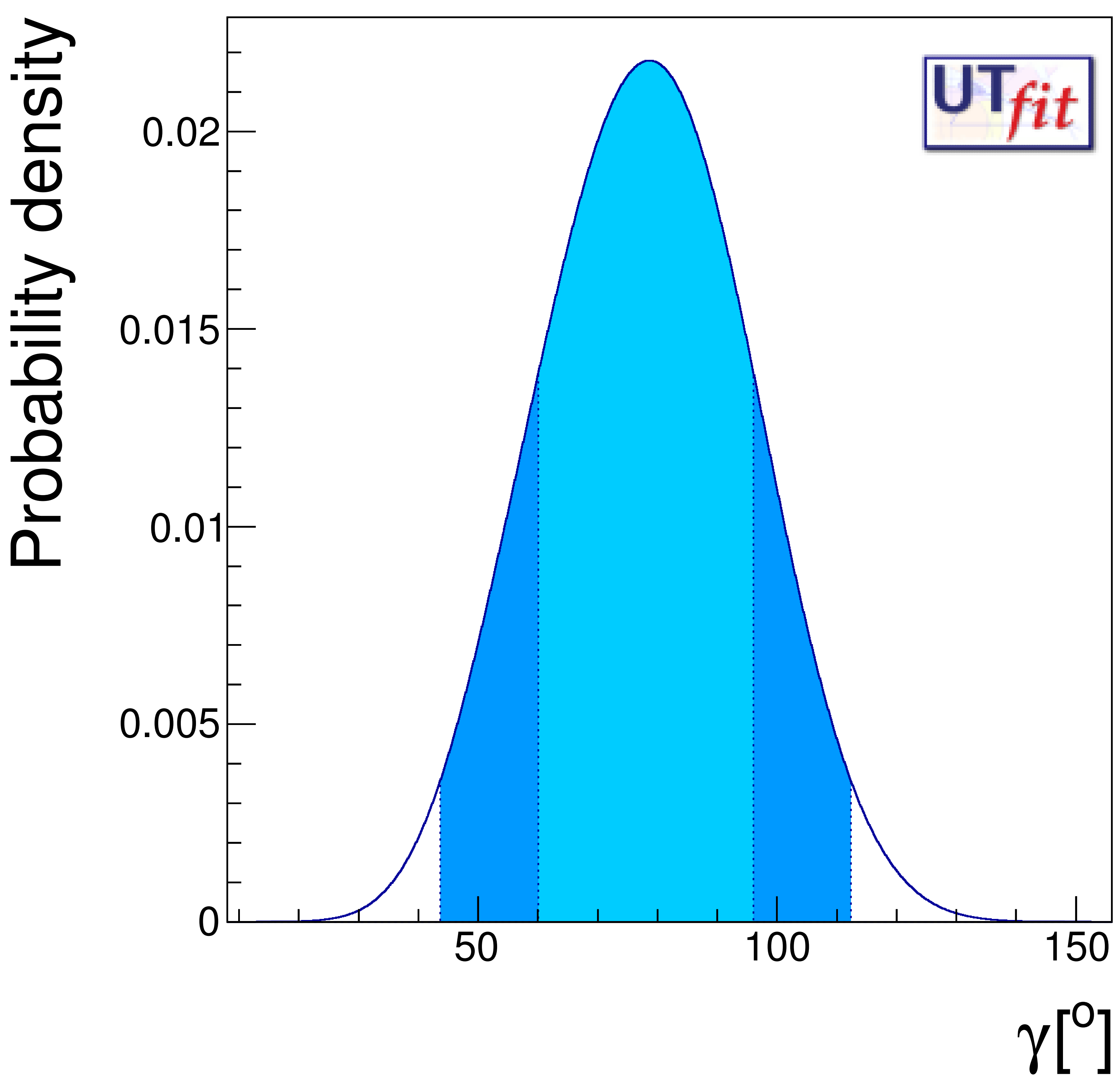
CKM angle γ with subsets of beauty observables

Charged B inputs only



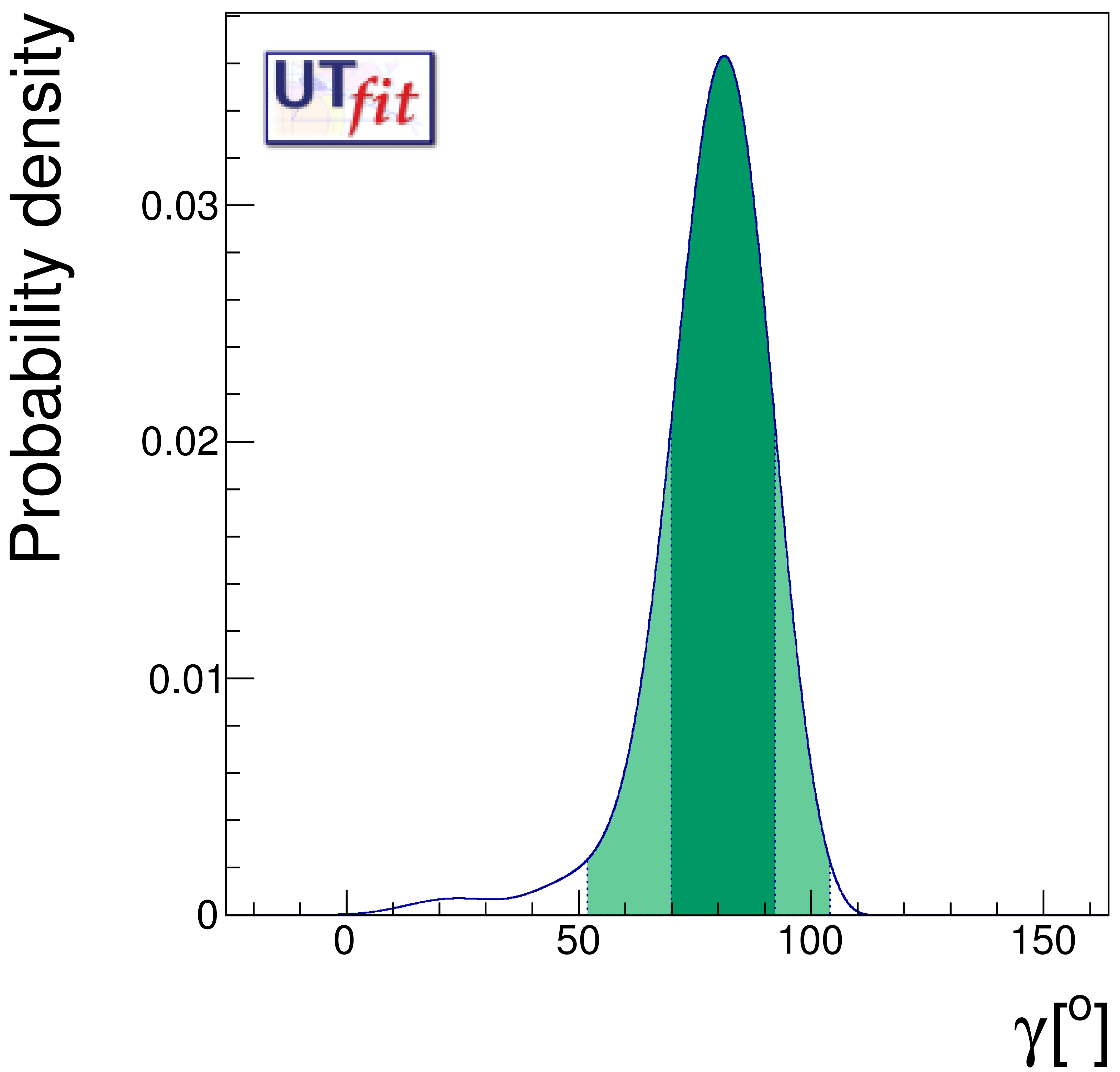
$$\gamma = (63.5 \pm 3.5)^\circ$$

Neutral B_s inputs only



$$\gamma = (78 \pm 18)^\circ$$

Neutral B_d inputs only

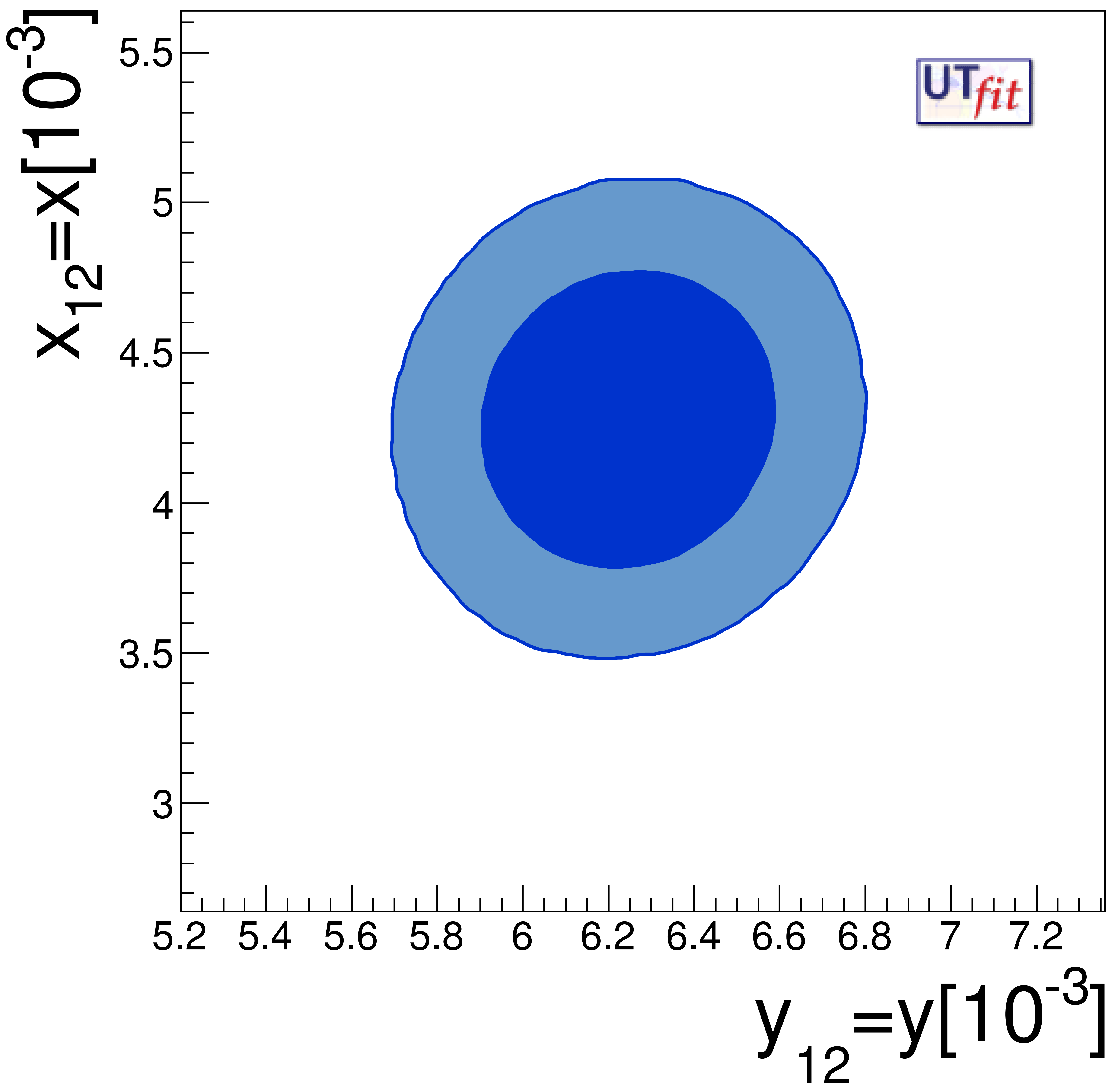


$$\gamma = (81 \pm 11)^\circ$$

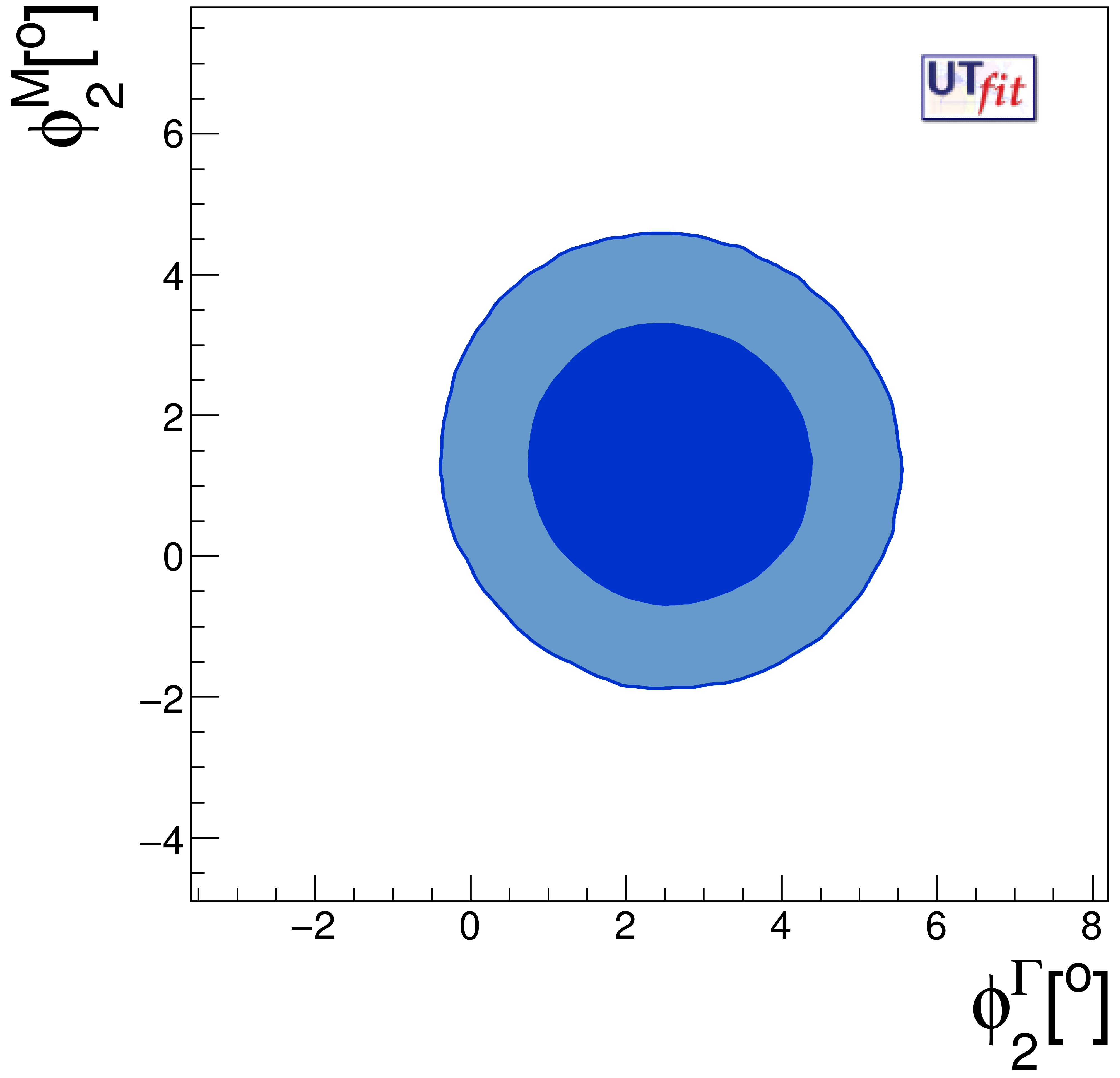
Charm mixing parameters

FIT RESULTS

$$x_{12} = (4.28 \pm 0.32)‰$$
$$y_{12} = (6.24 \pm 0.23)‰$$

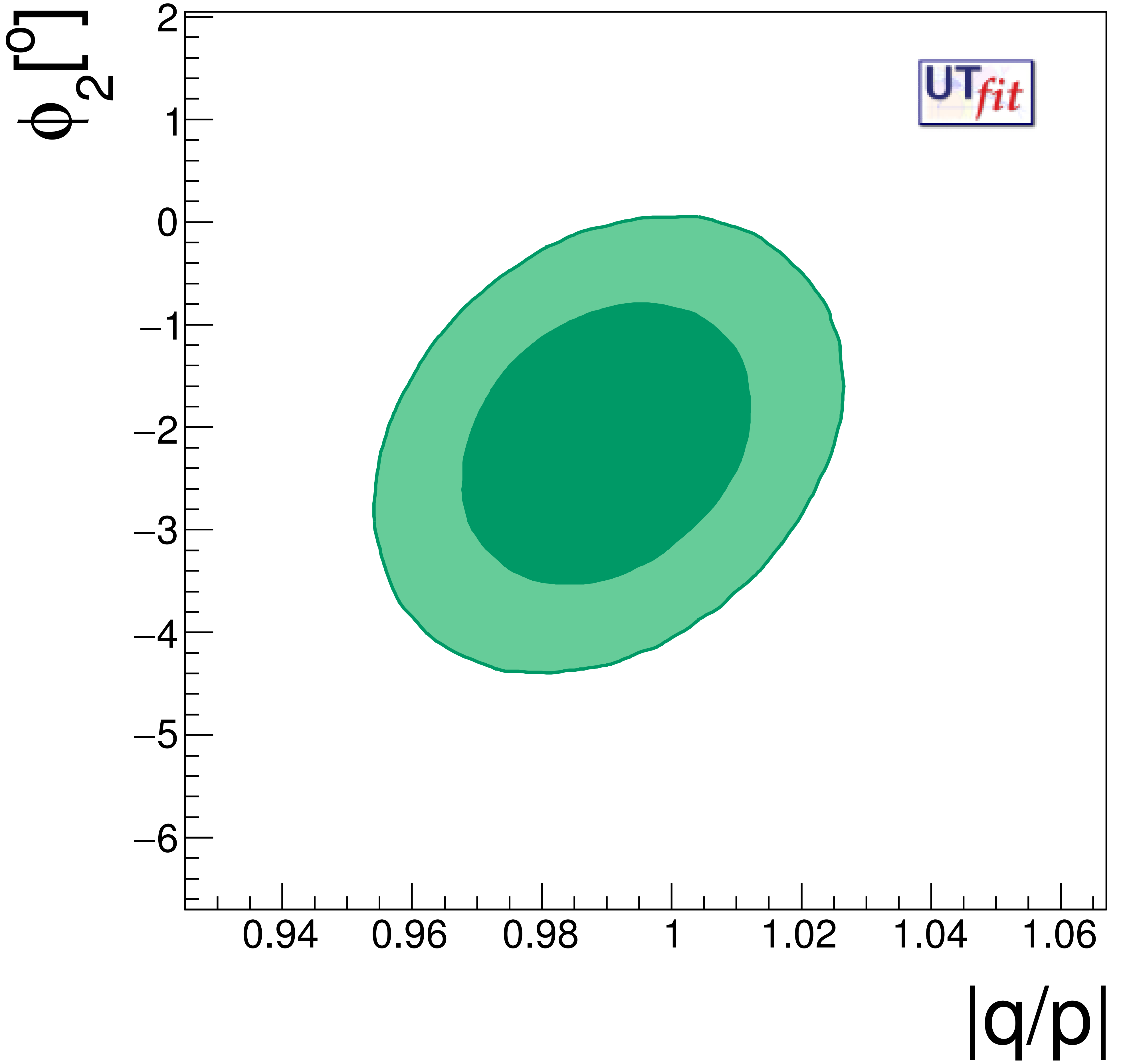


CPV parameters



FIT RESULTS (familiar formalism)

$\phi_2 = (-2.15 \pm 0.90)^\circ,\ |q/p| = 0.990 \pm 0.015$



FIT RESULTS (Kagan - Silvestrini)

$\phi_2^M = (1.3 \pm 1.3)^\circ,\ \phi_2^\Gamma = (2.6 \pm 1.2)^\circ$

Conclusions

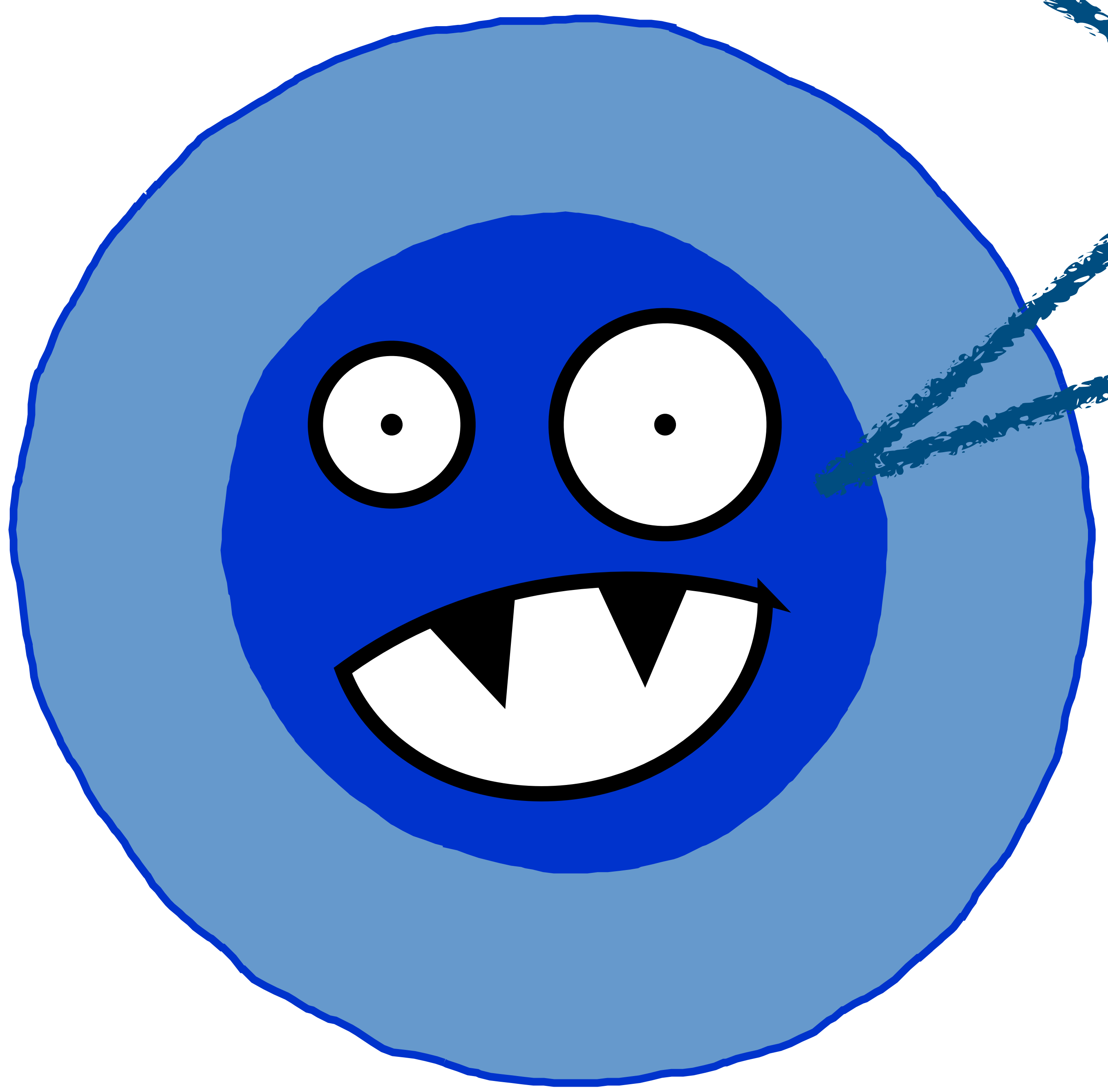
The results of the **combination** of charm and beauty observables have shown that:

- The parameters are **compatible** with the latest LHCb frequentist combination.
- The uncertainties on $\phi_2^{M,\Gamma}$ are still an **order of magnitude grater** than the estimates from the SM U-spin decomposition $\phi_2^{M,\Gamma} \approx 0.13°$.
- The estimate of γ from neutral B meson observables is 4/5 less precise than the one obtained from charged mesons. **(DIFFICULT TO CHECK CONSISTENCY)**

Interesting prospects for the future

- Waiting for the **next generation of experiments** (LHCb upgrades, Belle-II).
- Finding an efficient way to **compute** $\phi_2^{M,\Gamma}$ from first principles in the SM.
- **New processes for neutral B mesons** to improve the precision on γ (e.g. $B_q^0 \to D\phi$).

Thank you for your attention!



If you have any remark, you can find me around or you can reach me at
roberto.dipalma@uniroma3.it

Backup slides

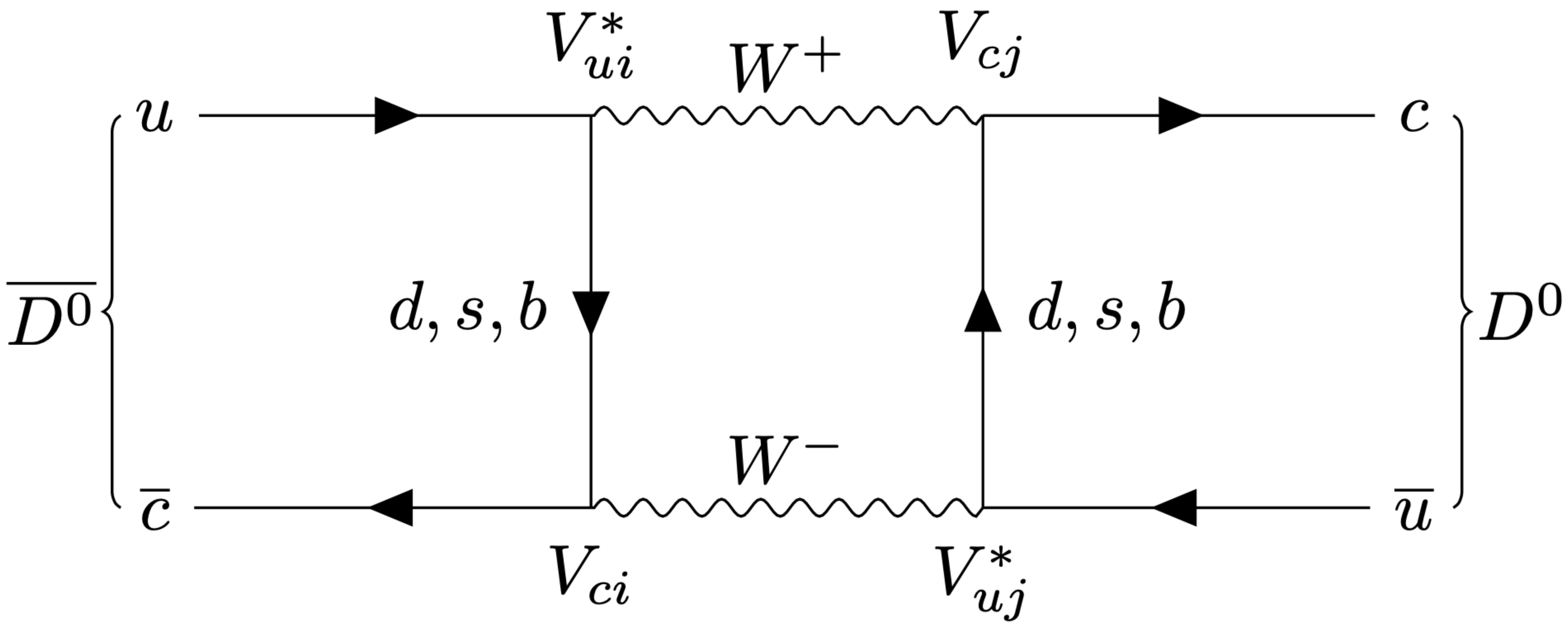
Long and short distance contributions

Neglecting external momenta

$$\frac{p^2 \approx \mathcal{O}(m_c^2)}{m_i^2} << 1$$

$i = b$

Generating a local $\Delta C = 2$ operator



&

$(\lambda^b_{uc} m_b)^2 \approx (\lambda^5 m_b)^2$

$i = d, s$

Generating long distance contributions

$\propto (\lambda^s_{uc} m_s)^2 \approx (\lambda m_s)^2$

$$9 \times 10^{-3} \times \left[\left(\frac{\lambda}{0.22} \right)^4 \times \frac{m_b}{4\text{GeV}} \frac{0.1\text{GeV}}{m_s} \right]^2$$

U-spin decomposition

$$\Gamma_2 = (\bar{s}s - \bar{d}d)^2 = \mathcal{O}(\epsilon^2) \qquad \Gamma_0 = (\bar{s}s + \bar{d}d)^2 = \mathcal{O}(1)$$

$$\Gamma_1 = (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) = \mathcal{O}(\epsilon)$$

$$\lambda^s_{uc} - \lambda^d_{uc} \approx 0.44 - i1.2 \times 10^{-4} \qquad \lambda^b_{uc} \approx (5.7 + i12) \times 10^{-5}$$

$$\Gamma^{SM}_{12} = \frac{(\lambda^s_{uc} - \lambda^d_{uc})^2}{4} \Gamma_2 \times$$

→ **Dominant contribution**

$$\left[1 + (0.86 + i1.8) \times 10^{-3} \left(\frac{0.3}{\epsilon}\right) + (-6.4 + i7.8) \times 10^{-7} \left(\frac{0.3}{\epsilon}\right)^2\right]$$

CPV phases

$$\phi_2^X = \arg\left[\frac{X_{12}}{X_2(\lambda_{uc}^s - \lambda_{uc}^d)^2/4}\right]\Bigg|_{X=M,\Gamma}$$

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$$\delta\phi_f = \phi_f^X - \phi_2^X\Big|_{X=M,\Gamma}$$

CP eigenstates

$$\delta\phi_{f_{CP}} = \mathcal{O}\left(\frac{\lambda_{uc}^b \sin(\gamma)}{\lambda}\right)$$

Misalignments

CF/DCS decays

$$\delta\phi_f = \mathcal{O}\left(\frac{\lambda_{uc}^b}{\lambda_{uc}^d}\right)^2 \approx 5.5 \times 10^{-6}$$

SM rough estimates

$$\phi_2^\Gamma\Big|_{SM} = \arg\left[\frac{2\lambda_{uc}^b}{\lambda_{uc}^s - \lambda_{uc}^d}\frac{\Gamma_1}{\Gamma_2}\right] = \arg\left[-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \times \left(\frac{1}{1 - \frac{V_{us}^* V_{cs}}{V_{ud}^* V_{cd}}}\right)\right] = \left|\frac{\lambda_{uc}^b}{\lambda_{uc}^d}\right| \sin(\gamma)\epsilon^{-1} \approx (2.2 \times 10^{-3}) \times \left[\frac{0.3}{\epsilon}\right]$$

Connecting the formalisms

F A M I L I A R

$$|x| = 1/\sqrt{2}\left[x_{12}^2 - y_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2\phi_{12}}\right]^{1/2} = x_{12} + \mathcal{O}(\phi_{12}^2)$$

$$y = 1/\sqrt{2}\left[y_{12}^2 - x_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2\phi_{12}}\right]^{1/2} = y_{12} + \mathcal{O}(\phi_{12}^2)$$

$$\left|\frac{q}{p}\right| = \left[\frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12}\sin\phi_{12}}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12}\sin\phi_{12}}\right]^{1/4} = 1 + \frac{x_{12}y_{12}}{x_{12}^2 + y_{12}^2}\sin\phi_{12} + \mathcal{O}(\phi_{12}^2)$$

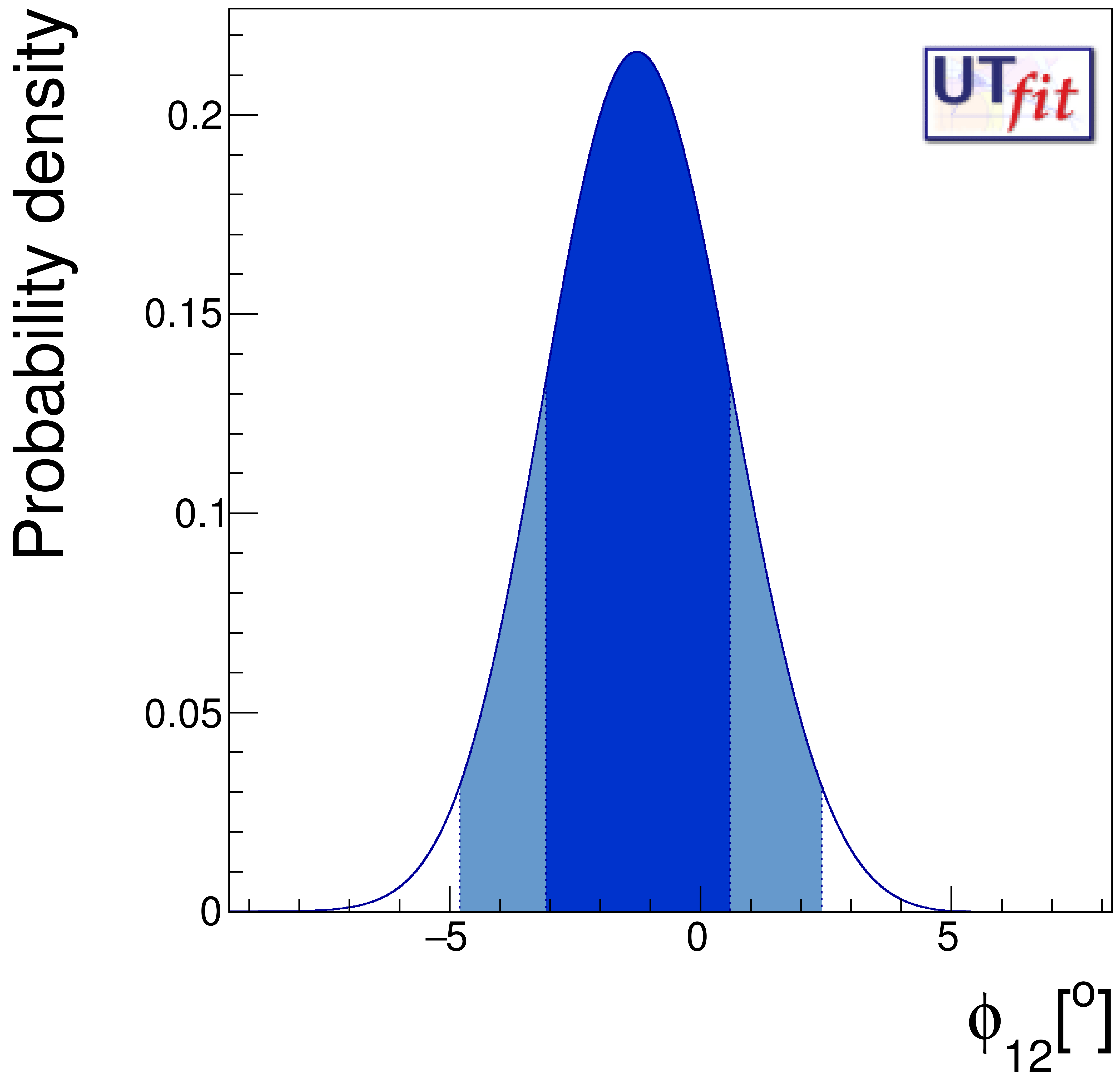
$$\tan(2\phi_{\lambda_f}) = -\frac{x_{12}^2 \sin 2\phi_f^M + y_{12}^2 \sin 2\phi_f^\Gamma}{x_{12}^2 \cos 2\phi_f^M + y_{12}^2 \cos 2\phi_f^\Gamma} \approx -\frac{x_{12}^2}{x_{12}^2 + y_{12}^2}\phi_f^M - \frac{y_{12}^2}{x_{12}^2 + y_{12}^2}\phi_f^\Gamma + \mathcal{O}(\phi_{12}^2)$$

K A G A N - S I L V E S T

CPV in pure mixing

FIT RESULT

$$\phi_{12} = (-1.3 \pm 1.8)^\circ$$



GLW/ADS Observables

(quasi-) GLW

$$\mathfrak{A}^h_{f_{CP}} = \frac{\Gamma(B \to [f_{CP}]_D h) - \Gamma(\bar{B} \to [f_{CP}]_D \bar{h})}{\Gamma(B \to [f_{CP}]_D h) + \Gamma(\bar{B} \to [f_{CP}]_D \bar{h})}$$

$$\Omega^h_{f_{CP}} = \frac{\Gamma(B \to [f_{CP}]_D h) + \Gamma(\bar{B} \to [f_{CP}]_D \bar{h})}{\Gamma(B \to [f]_D h) + \Gamma(\bar{B} \to [\bar{f}]_D \bar{h})}$$

$$\tilde{\mathfrak{R}}^{h_1/h_2}_{f_{CP}}(f) = \frac{\Omega^{h_1}_{f_{CP}}(f)}{\Omega^{h_2}_{f_{CP}}(f)}$$

$$\mathfrak{R}^h_{f_{CP}}(f) = \frac{\mathscr{B}(\overline{D^0} \to \bar{f})}{\mathscr{B}(\overline{D^0} \to f_{CP})}\Omega^h_{f_{CP}}(f)$$

(quasi-) ADS

$$A^{sup,fav}_h(f) = \frac{\Gamma(B \to [\bar{f}(f)]_D h) - \Gamma(\bar{B} \to [f(\bar{f})]_D \bar{h})}{\Gamma(B \to [\bar{f}(f)]_D h) + \Gamma(\bar{B} \to [f(\bar{f})]_D \bar{h})}$$

$$R^{ADS}_h(f) = \frac{\Gamma(B \to [\bar{f}]_D h) + \Gamma(\bar{B} \to [f]_D \bar{h})]}{\Gamma(B \to [f]_D h) + \Gamma(\bar{B} \to [\bar{f}]_D \bar{h})}$$

$$R^{sup,fav}_{h_1/h_2}(f) = \frac{\Gamma(B \to [\bar{f}(f)]_D h_1) + \Gamma(\bar{B} \to [f(\bar{f})]_D \bar{h}_1)}{\Gamma(B \to [\bar{f}(f)]_D h_2) + \Gamma(\bar{B} \to [f(\bar{f})]_D \bar{h}_2)}$$

$$R^h_+(f) = \frac{\Gamma(\bar{B} \to [f]_D \bar{h})}{\Gamma(\bar{B} \to [\bar{f}]_D \bar{h})} \qquad R^h_-(f) = \frac{\Gamma(B \to [\bar{f}]_D h)}{\Gamma(B \to [f]_D h)}$$