Towards the physical charmonium spectrum with improved distillation

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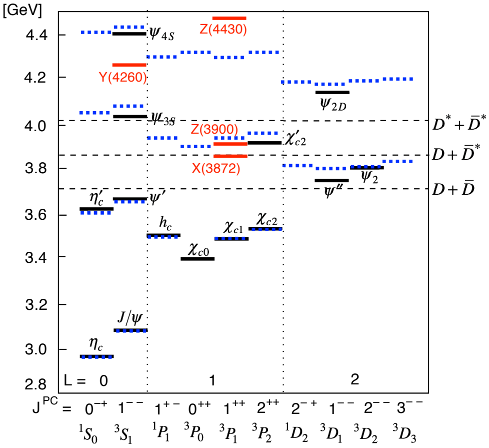
FOR 5269

Goal

Map out the physical charmonium spectrum using improved methods from lattice QCD.



Hosaka *et al.*, PTEP 2016 (2016) 6, 062C01.

Methodology

Ensemble with

- 1 physical charm quark + 3 light quarks at SU(3) flavor symmetric point, sum of masses as in nature.
- Lattice size $48^3 \times 144$ at $m_\pi \approx 420$ MeV, lattice spacing $a \approx 0.043$ fm.

We measure Euclidean temporal correlation functions $C(t) = \left\langle \mathcal{O}(t)\bar{\mathcal{O}}(0) \right\rangle$:

- Meson operators $\mathcal{O}(t) = \bar{q}(t)\Gamma q(t)$ at zero spatial momentum.
- Γ related to J^{PC}, e.g $\Gamma = \gamma_5, \gamma_i, \nabla_i, \ldots$
- $C(t) \overset{t\to\infty}{\approx} A_0 e^{-E_0 t}$, E_0: energy of the lightest state in symmetry channel.
- $am_{eff}(t) = \log\left(\frac{C(t)}{C(t+a)}\right) \to$ Effective mass in lattice units.

Obstacles

Computational: Costs and sizes!

! $C(t)$ involves terms $Tr\left(\Gamma D^{-1}[t,0]\Gamma D^{-1}[0,t]\right)$, $Tr\left(\Gamma D^{-1}[t,t]\right)$.

- Dirac operator D of size $\approx 10^7 \times 10^7$ and **very** sparse.
- Traces are often **stochastically** estimated.
- Need **many** solves of the form $Dx = b$

! $C(t)$ is approximated via Monte-Carlo average so **large statistics** are important!

Physical:

! $C(t) = A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + \ldots \rightarrow$ Often need **large** t to reliably extract E_0.

! For some operators, $C(t)$ exhibits a **signal-to-noise problem** $\rightarrow$ Large t are not always within reach.

Tools

Distillation. M. Peardon *et al.* Phys. Rev. D 80, 054506 (2009).

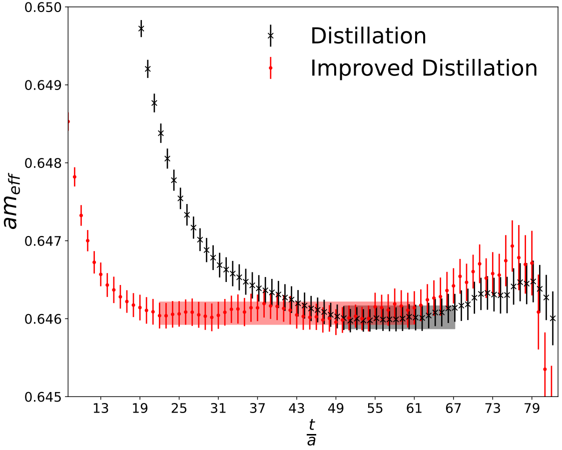
- ✓ $\psi(t) \to V[t]V[t]^\dagger\psi(t)$. $V[t]$: Lowest eigenvectors of 3D covariant Laplacian $\nabla^2[t]$. Smooth, gauge-covariant fields.
- ✓ $D^{-1}[t,0] \to \tau[t,0] = V[t]^\dagger D^{-1} V[0]$ "Perambulators". Exact calculation and storage are feasible.
- ✓ $\Gamma \to \Phi[t] = V[t]^\dagger \Gamma V[t]$ "Elementals". Wide variety of Γ at fixed inversion cost.

Improved distillation J. A. Urrea-Niño, F. Knechtli, T. Korzec & M. Peardon. Phys. Rev. D 106, 034501 (2022)

- ✓ $\Phi[t]_{\substack{ij \\ \alpha\beta}} = v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t] \to \tilde{\Phi}[t]_{\substack{ij \\ \alpha\beta}} = \tilde{f}\left(\lambda_i[t], \lambda_j[t]\right) v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t]$
- ✓ $\lambda_i[t]$: eigenvalues of $\nabla^2[t]$.
- ✓ **Optimal** $\tilde{f}\left(\lambda_i[t], \lambda_j[t]\right)$ for each Γ and excitations are calculated. GEVP method from lattice QCD.
- ✓ Same computational advantages of Distillation **plus** improvement at very little additional cost.

Improvement Pt. 1

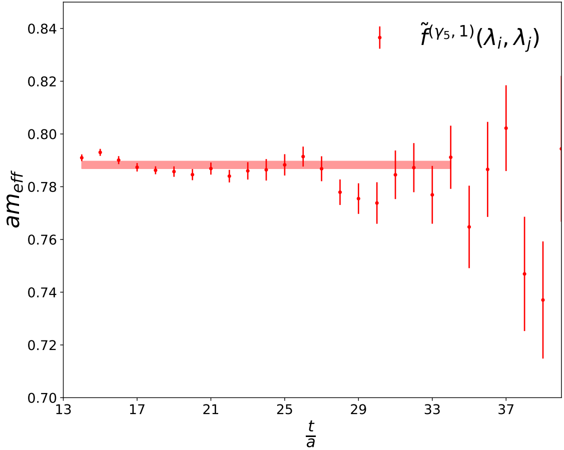
E.g Ground state $J^{PC} = 0^{-+}$ $(\Gamma = \gamma_5)$, **omitting** quark-anti-quark annihilation effects.



Earlier/Longer effective mass plateau. Less excited-state contamination.

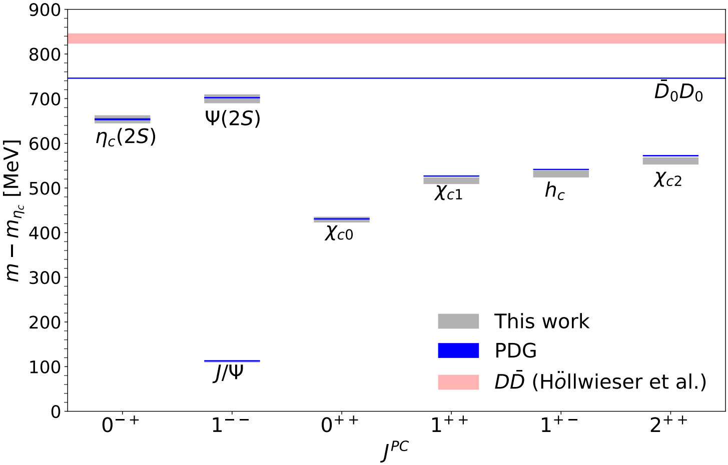
Improvement Pt. 2

First excited state of $J^{PC} = 0^{-+}$ ($\Gamma = \gamma_5$).



Distillation would require different Γ to access this state via GEVP.

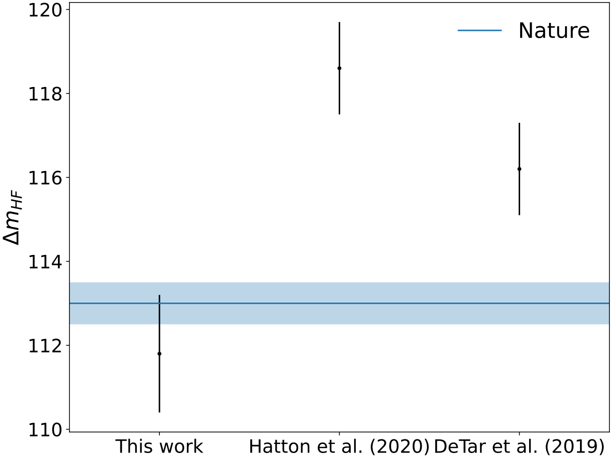
Charmonium spectrum



Good agreement with physical charmonium!

Charmonium hyperfine splitting

$$\Delta m_{HF} = m_{J/\Psi} - m_{\eta_c}$$



✓ Statistical precision is comparable to other lattice calculations.

Charmonium mass splittings

$$\Delta m_{1S-1P} = \frac{1}{9}\left(m_{\chi_{c0}} + 3m_{\chi_{c1}} + 5m_{\chi_{c2}}\right) - \frac{1}{4}\left(m_{\eta_c} + 3m_{J/\Psi}\right)$$

$$\Delta m_{SO} = \frac{1}{9}\left(5m_{\chi_{c2}} - 3m_{\chi_{c1}} - m_{\chi_{c0}}\right)$$

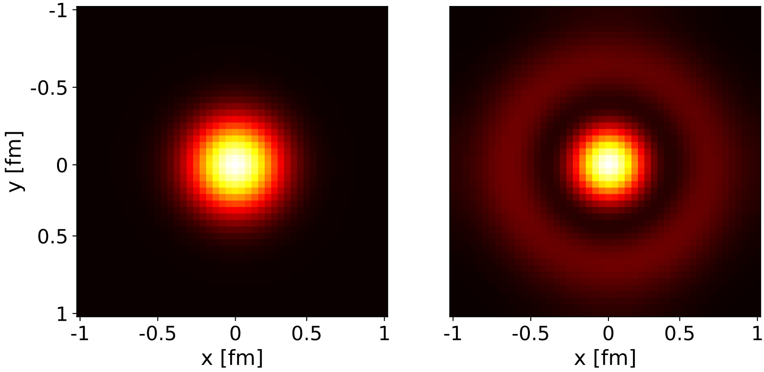
$$\Delta m_{\text{tensor}} = \frac{1}{9}\left(3m_{\chi_{c1}} - m_{\chi_{c2}} - 2m_{\chi_{c0}}\right)$$

$$\Delta m_{1\text{PHF}} = \frac{1}{9}\left(m_{\chi_{c0}} + 3m_{\chi_{c1}} + 5m_{\chi_{c2}}\right) - m_{h_f}$$

Splitting	This work (MeV)	PDG (MeV)	DeTar *et al.* (MeV)
Δm_{1P-1S}	447.3(5.5)	456.64(14)	462.2(4.5)
Δm_{SO}	43.93(87)	46.60(8)	46.6(3.0)
Δm_{tensor}	14.43(41)	16.27(7)	17.0(2.3)
$\Delta m_{1\text{PHF}}$	-0.2(1.6)	-0.09(14)	-6.1(4.2)
Δm_{HF-1}	45.9(1.8)	48(1)	

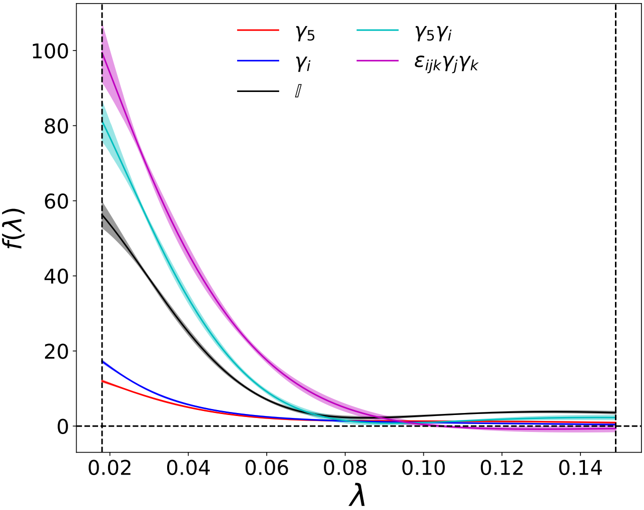
Spatial Profiles of Charmonium

Effect of $V[t]\tilde{\Phi}[t]V[t]^\dagger$ on a point-source $\rightarrow$ Spatial structure arises.



- $J^{PC} = 0^{-+}$ $(\Gamma = \gamma_5)$ agrees with S-wave behavior. $S = 0, L = 0$
- Spatial volume $= 2.0592^3$ fm^3. States are well-contained.

Meson Profiles of Charmonium



Different Γ have different profiles! None is a constant.

Conclusions and Outlook

- Optimal distillation profiles substantially **reduce** excited-state **contamination**.
- The measured low-lying charmonium spectrum and mass splittings **agree** with nature.
- Statistical precision is **comparable** to state-of-the-art lattice calculations, if not better.

Charm annihilation contributions to correlation functions were omitted for now, however:

- Improved distillation is applicable for iso-scalar operators. **See talk by Tomasz Korzec.**
- Also for mixing with glueballs and light hadrons. **See talk by Roman Höllwieser.**

Thank you for your attention!

Details on the improvement

$V[t]$ are the low eigenvectors of the 3D covariant Laplacian

$$\nabla^2[t]_{\vec{x}\vec{y}} = -6\delta_{\vec{x}\vec{y}} + \sum_{i=1}^{3} U_i(\vec{x},t)\delta_{\vec{x}+\hat{\imath},\vec{y}} + U_i(\vec{x}-\hat{\imath},t)^\dagger \delta_{\vec{x}-\hat{\imath},\vec{y}}$$

Distillation has *elementals*

$$\Phi[t] = V[t]^\dagger \Gamma V[t] \to \Phi[t]_{\substack{ij\\ \alpha\beta}} = v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t]$$

while the improved variant has

$$\tilde{\Phi}[t] = V[t]^\dagger \Gamma V[t] \circledast F[t] \to \tilde{\Phi}[t]_{\substack{ij\\ \alpha\beta}} = v_i[t]^\dagger \Gamma_{\alpha\beta} v_j[t] \tilde{f}\left(\lambda_i[t], \lambda_j[t]\right)$$

The optimal meson profile modulates the coupling of the vectors with the chosen Γ. It is in general **not** separable.

The GEVP

Build correlation matrix

$$C_{ij}(t) = \left\langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0) \right\rangle$$

and solve the **G**eneralized **E**igen**v**alue **P**roblem

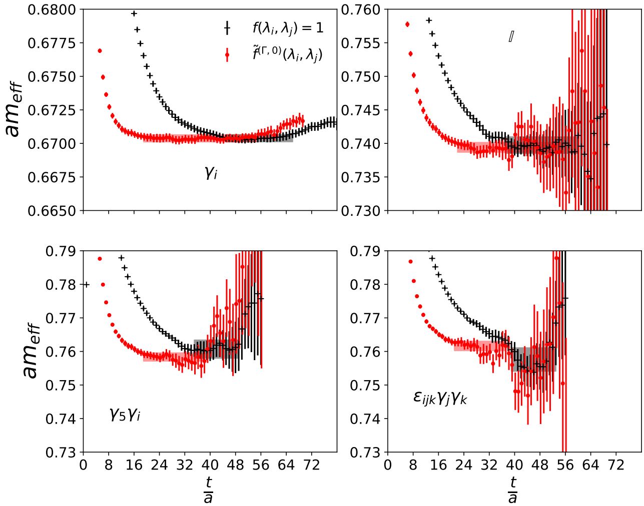
$$C(t)u_i(t,t_0) = \rho_i(t,t_0)C(t_0)u_i(t,t_0)$$

for fixed t_0 and $t > t_0$. In the appropriate time interval:

- $\rho_i(t,t_0) \approx B_0 e^{-E_0 t}$.
- $u_i(t,t_0)$ are approximately constant.
- $\tilde{\mathcal{O}}_i = \sum_k u_i^{(k)} \mathcal{O}_k$ corresponds to state that best approximates the $i-th$ energy eigenstate.

See M.Lüscher and U. Wolff, Nucl. Phys. B339, 222 (1990), Blossier *et al.*, J. High Energy Phys. 2009, 094.

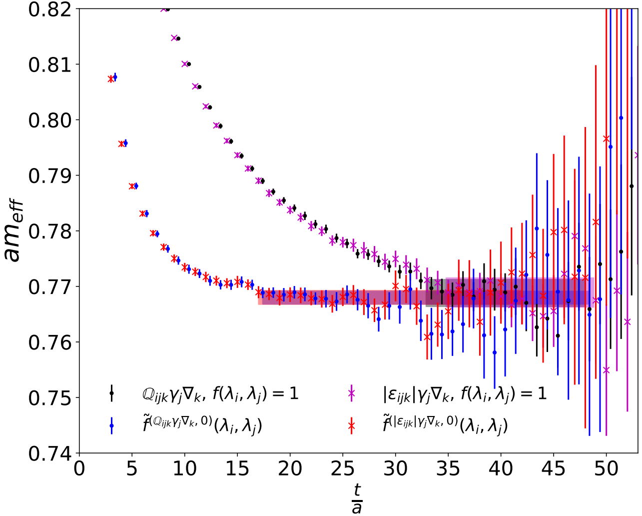
Other choices of Γ



The method also extends to derivative-based choices of Γ.

2^{++} Assignment

$2^{++} \rightarrow E^{++} \oplus T_2^{++}$. Degeneracy in $a \rightarrow 0$ limit.



References

Hatton *et al.*, Phys. Rev. D 102, 054511.

- $2+1+1$ and $1+1+1+1$ HISQ ensembles.
- Charm tuned to recover physical J/Ψ.
- Some effects of QED are also studied.
- ...

DeTar *et al.*, Phys. Rev. D. 99, 034509.

- $2+1$ asqtd ensembles for u/d/s.
- Charm included via matching physical D_s.
- ...

Höllwieser *et al.*, The European Physical Journal C 80, 349.

- Generation of the ensemble used here.
- Charmonium spectrum + D meson
- ...