Resummation and Renormalization of Kinematical Effects in χ_c and χ_b Hadroproduction

Hee Sok Chung
Korea University

Based on JHEP07(2023)007
arXiv:2303.17240 [hep-ph]

11th International Workshop on Charm Physics (CHARM 2023)
July 20, 2023

P-wave Production in NRQCD

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- P-wave (χ_c, χ_b) production at LO in v :

$$\sigma[\chi_{QJ}(P)] = (2J+1)\left(c_{^3P_J^{[1]}}(P)\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + c_{^3S_1^{[8]}}(P)\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle\right)$$

(Q=c **or** b)

short-distance coefficients, generally available to NLO

- $c_{^3P_J^{[1]}}(P)$ and $c_{^3S_1^{[8]}}(P)$ describe perturbative production of $Q\bar{Q}$. Matrix elements (ME) $\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle$ and $\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$ describe nonperturbative evolution of $Q\bar{Q}$ into quarkonium.

- Color-octet matrix elements are obtained from fits to data, which depend on normalization and p_T dependence of cross section.

- p_T shapes of cross sections come from $c_{^3P_J^{[1]}}(P)$ and $c_{^3S_1^{[8]}}(P)$. Fixed-order calculations are known to have difficulty describing cross sections over a wide range of p_T.

P-wave Production in NRQCD

- *P*-wave (χ_c, χ_b) production at LO in v :

$$\sigma[\chi_{QJ}(P)] = (2J+1)\left(c_{^3P_J^{[1]}}(P)\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + c_{^3S_1^{[8]}}(P)\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle\right)$$

- $\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle$: color-singlet $Q\bar{Q}$ evolve into χ_Q **(with or without soft radiation)**
  $\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$: color-octet $Q\bar{Q}$ evolve into χ_Q by soft radiation.

- Color-octet $Q\bar{Q}$ can also evolve into color-singlet $Q\bar{Q}$ by soft radiation before evolving into χ_Q.
  Two channels mix by soft gluon emission due to renormalization:

$$\left(\frac{d}{d\log\Lambda}c_{^3P_J^{[1]}}(P)\right)\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + c_{^3S_1^{[8]}}(P)\left(\frac{d}{d\log\Lambda}\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle\right) = 0$$

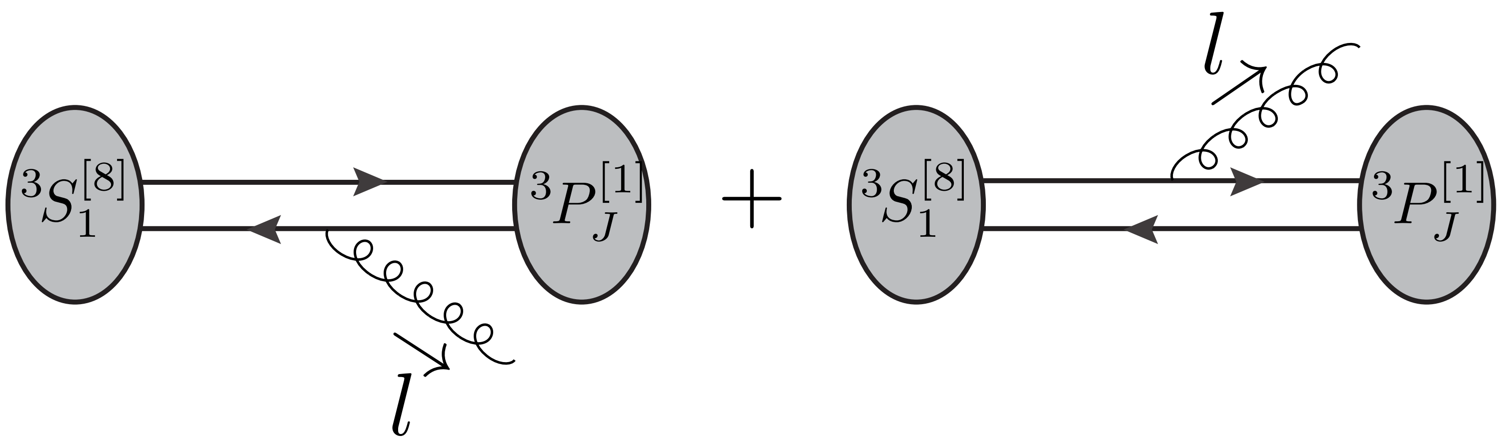
scale dependence of short-distance coefficient

scale dependence of renormalized color-octet ME

- Same form of mixing happens for *S*-wave quarkonium production (ψ, Υ) between color-octet ^3P_J and $^3S_1$ states

Mixing in *P*-wave Production

- Mixing in dimensional regularization $d=4-2\epsilon$:



$$\propto \int_0^\infty \frac{d|\boldsymbol{l}|}{|\boldsymbol{l}|^{1+2\epsilon}} c_{^3S_1^{[8]}}(P+l)$$

vanishes for $l>l_{\rm max}$

- $$\int_0^{|\boldsymbol{l}_{\rm max}|} \frac{d|\boldsymbol{l}|}{|\boldsymbol{l}|^{1+2\epsilon}} c_{^3S_1^{[8]}}(P+l) = -\frac{|\boldsymbol{l}_{\rm max}|^{-2\epsilon}}{2\epsilon_{\rm IR}} c_{^3S_1^{[8]}}(P) + \int_0^{|\boldsymbol{l}_{\rm max}|} \frac{d|\boldsymbol{l}|}{|\boldsymbol{l}|} \left[c_{^3S_1^{[8]}}(P+l) - c_{^3S_1^{[8]}}(P)\right] + O(\epsilon)$$

IR pole at $l=0$ produces scale dependence

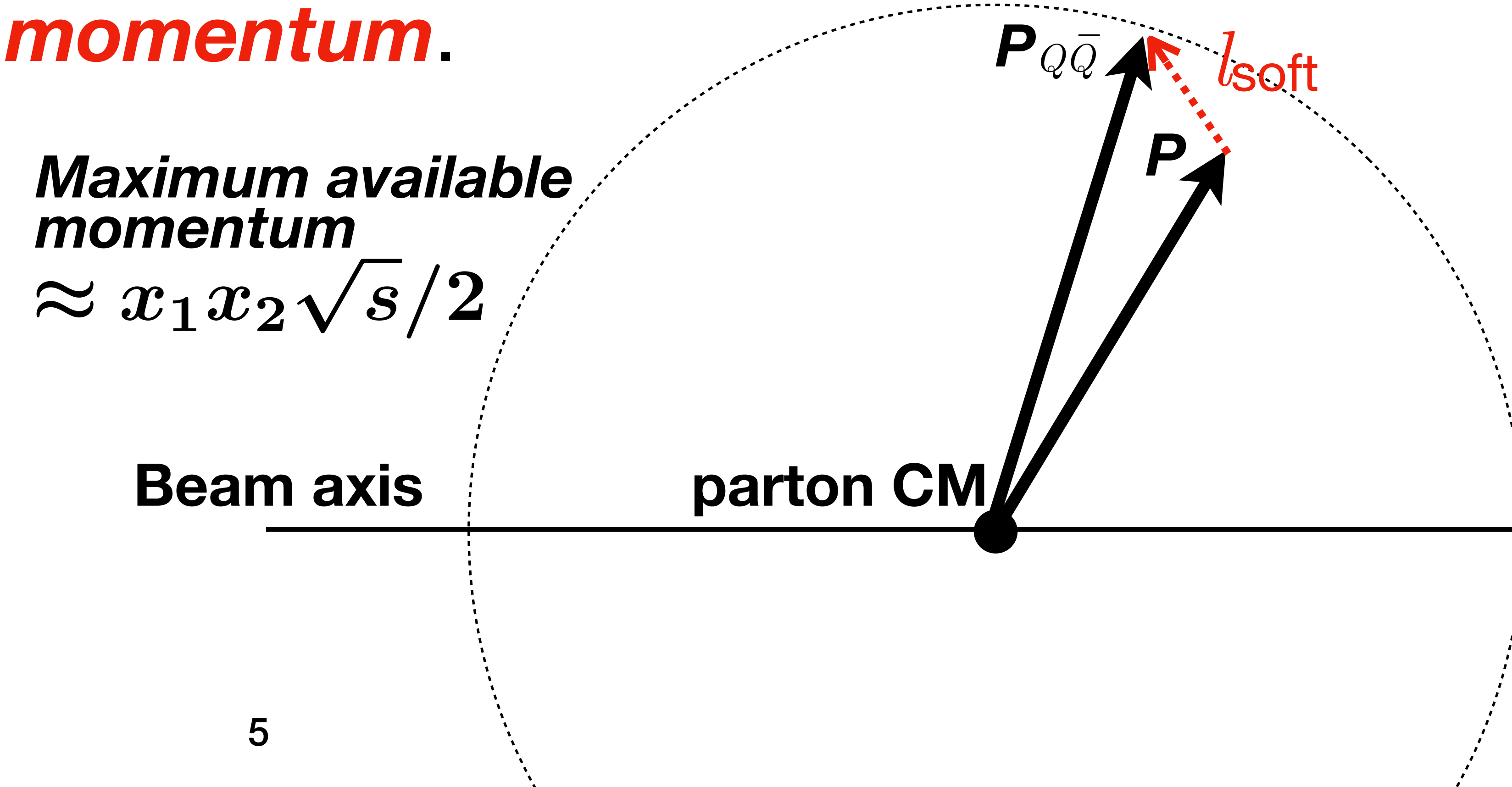
plus distribution, singular at $l=0$

- Gluon momentum $l>0$ is included in the perturbative short-distance coefficient, while only $l=0$ is included in the nonperturbative ME. *However, nonperturbative soft gluons can have **nonzero momentum**.*

Production Kinematics

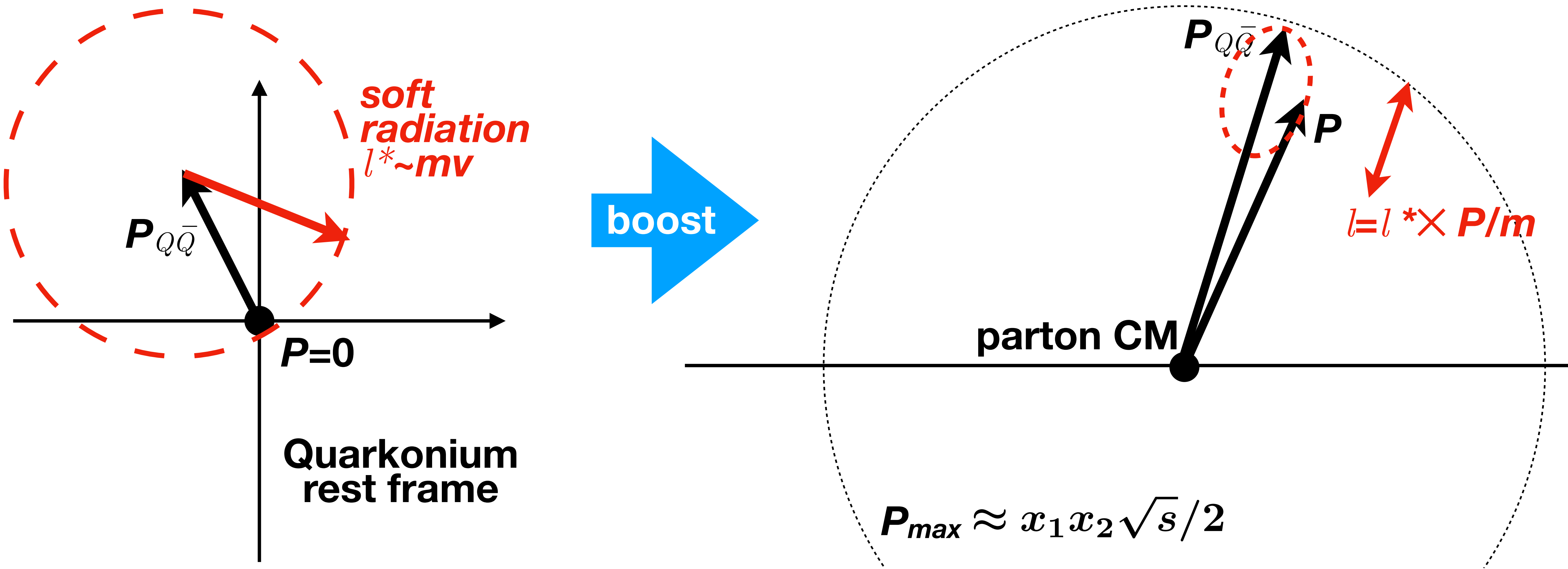
- *Nonzero soft gluon momentum* implies that *quarkonium momentum is smaller than $Q\bar{Q}$ momentum*

- In large-p_T hadroproduction, $|\boldsymbol{P}_{Q\bar{Q}}| \lesssim x_1 x_2 \sqrt{s}/2$
  Cross section is sensitive to small changes in Bjorken x.
  $Q\bar{Q}$ cross section is singular at maximum ***P*** ($l_{\rm soft}$=0).
  Hence, quarkonium cross section can be sensitive to soft gluon momentum.



Production Kinematics

- Soft radiation is "soft ($\sim mv$)" in the quarkonium rest frame. Need to boost from quarkonium rest frame to CM frame.



- Soft radiation has *no preferred direction in the* ***P=0*** *rest frame*, but after a large boost ($P \gg m_\chi$) the only relevant direction of soft radiation is ***lightlike*** and ***anti-parallel*** to ***P***.
This effect can be resummed!

Shape Functions

- Schematic form of lowest-dimensional NRQCD matrix elements, defined in quarkonium rest frame:

some combination of Pauli and color matrices, covariant derivatives

$$\langle \mathcal{O}^{\chi_Q}(\Gamma) \rangle = \langle \chi^\dagger \Gamma \psi \mathcal{P}_{\chi_Q} \psi^\dagger \Gamma \chi \rangle$$

- We can have operator matrix elements that read off the $Q\bar{Q}$ momentum in a specific direction l given by

$$\langle \chi^\dagger \Gamma \psi \mathcal{P}_{\chi_Q} (l \cdot D)^n \psi^\dagger \Gamma \chi \rangle$$

- These matrix elements are generated by the "shape function"

$$\mathcal{S}_\Gamma^{\chi_Q}(l_+) = \langle \chi^\dagger \Gamma \psi \mathcal{P}_{\chi_Q} \delta(l_+ - iD_+) \psi^\dagger \Gamma \chi \rangle$$

- In practice, we only need color-octet shape functions, because due to vacuum-saturation approximation, color-singlet shape functions are trivial : $\mathcal{S}_{\Gamma_{\rm singlet}}^{\chi_Q}(l_+) = \langle \mathcal{O}^{\chi_Q}(\Gamma_{\rm singlet}) \rangle \delta(l_+)$

Shape Function Formalism

- NRQCD formalism :

$$\sigma[\chi_{QJ}(P)] = (2J+1)\left(c_{{}^3P_J^{[1]}}(P)\langle\mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle + c_{{}^3S_1^{[8]}}(P)\langle\mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle\right)$$

- Shape function formalism (NRQCD with kinematical corrections) :

$$\sigma[\chi_{QJ}(P)] = (2J+1)\Big(s_{{}^3P_J^{[1]}}(P)\langle\mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle + \int_0^\infty dl_+ s_{{}^3S_1^{[8]}}(P+l)\mathcal{S}^{\chi_{Q0}}_{{}^3S_1^{[8]}}(l_+)\Big)$$

l=momentum lost by soft radiation

- Formally $\int_0^\infty dl_+ \mathcal{S}^{\chi_{Q0}}_{{}^3S_1^{[8]}}(l_+) = \langle\mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle$,

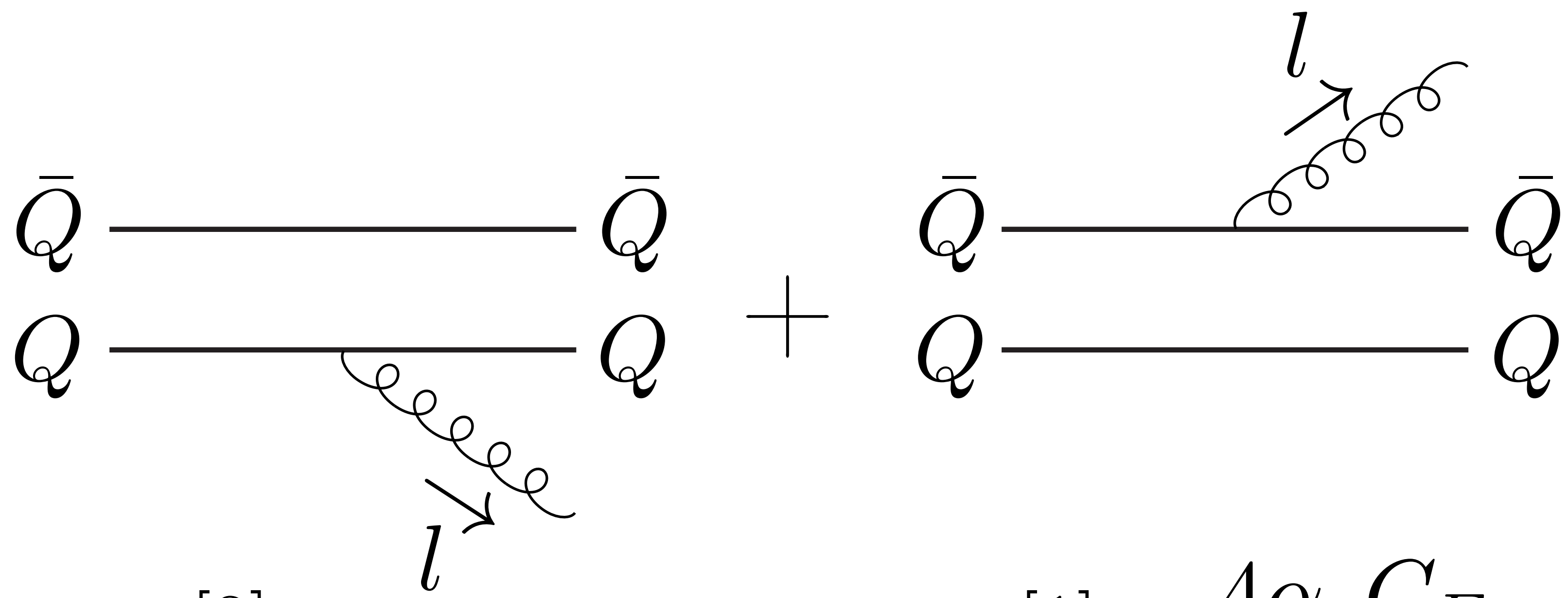
  but both sides are UV divergent and require renormalization.

- Knowledge of the nonperturbative shape function $\mathcal{S}^{\chi_{Q0}}_{{}^3S_1^{[8]}}(l_+)$ needed to compute cross sections.

Beneke, Rothstein, Wise, PLB408 (1997) 373
Fleming, Leibovich, Mehen, PRD68, 094011 (2003)

Renormalization

- Renormalization of color-octet matrix element



$$\langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle\Big|_{\rm UV} = \langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle \frac{4\alpha_s C_F}{3N_c \pi m^2}\int_0^\infty \frac{dl_+}{l_+^{1+2\epsilon}}$$

- Normalization of the shape function must reproduce this integral. This gives the asymptotic behavior at large l_+

$$\mathcal{S}^{\chi_{Q0}}_{{}^3S_1^{[8]}}(l_+)\Big|_{{\rm asy},\, d=4} = \langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle \times \frac{4\alpha_s C_F}{3N_c \pi m^2}\frac{1}{l_+}$$

Nonperturbative Shape Function

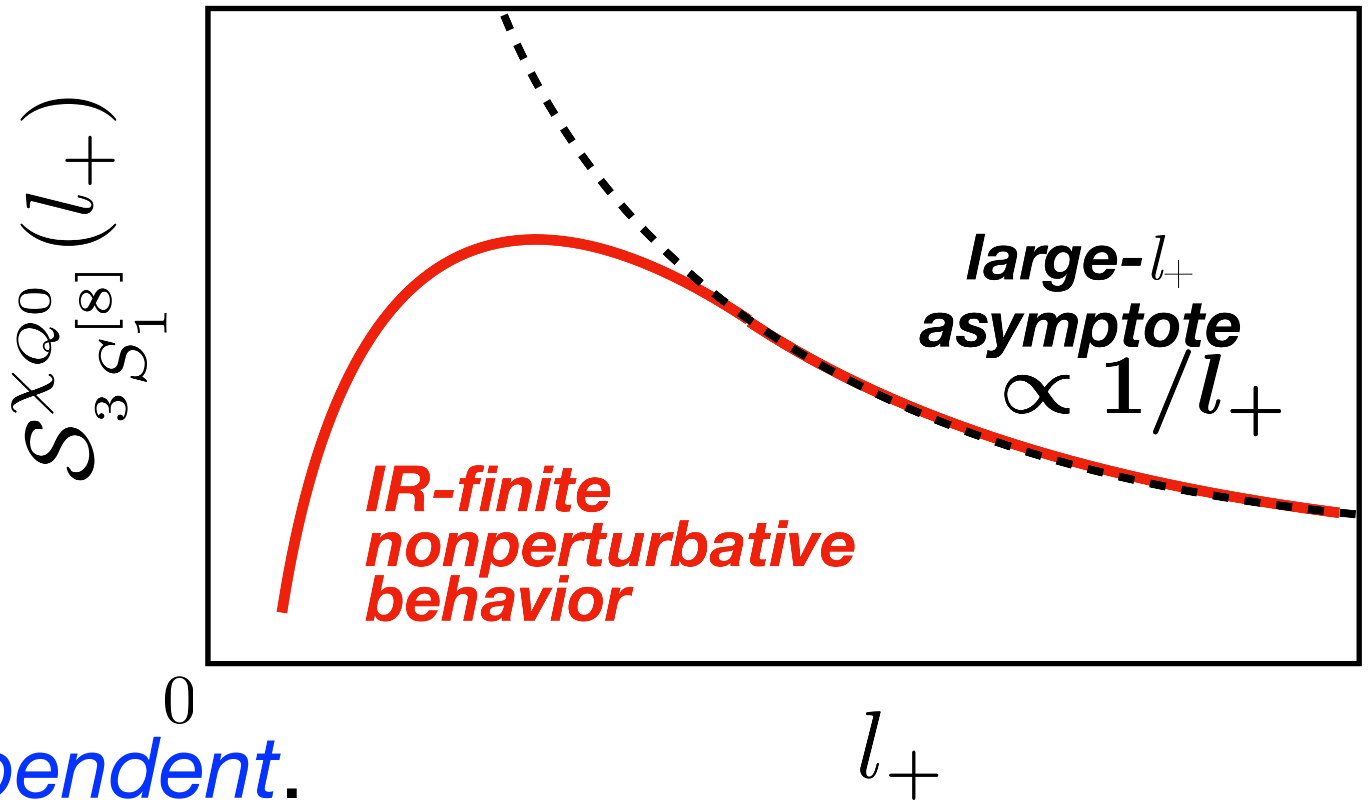
- *Asymptotic behavior* from *renormalization*:

$$\frac{d}{d\log\Lambda}\int_0^\Lambda dl_+ \mathcal{S}^{\chi_{Q0}}_{{}^3S_1^{[8]}}(l_+) = \frac{d}{d\log\Lambda}\langle\mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle^{(\Lambda)}|_{\rm renormalized}$$

- Nonperturbative normalization must be *IR finite*:

$$\int_0^\infty dl_+ \mathcal{S}^{\chi_{Q0}}_{{}^3S_1^{[8]}}(l_+) = \langle\mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle|_{\rm bare}$$

- Form of nonperturbative shape function is strongly constrained from ***renormalization*** and ***IR finiteness***.



- $l_+ \to 0$ behavior is *model dependent*.

Corrections to Cross Section

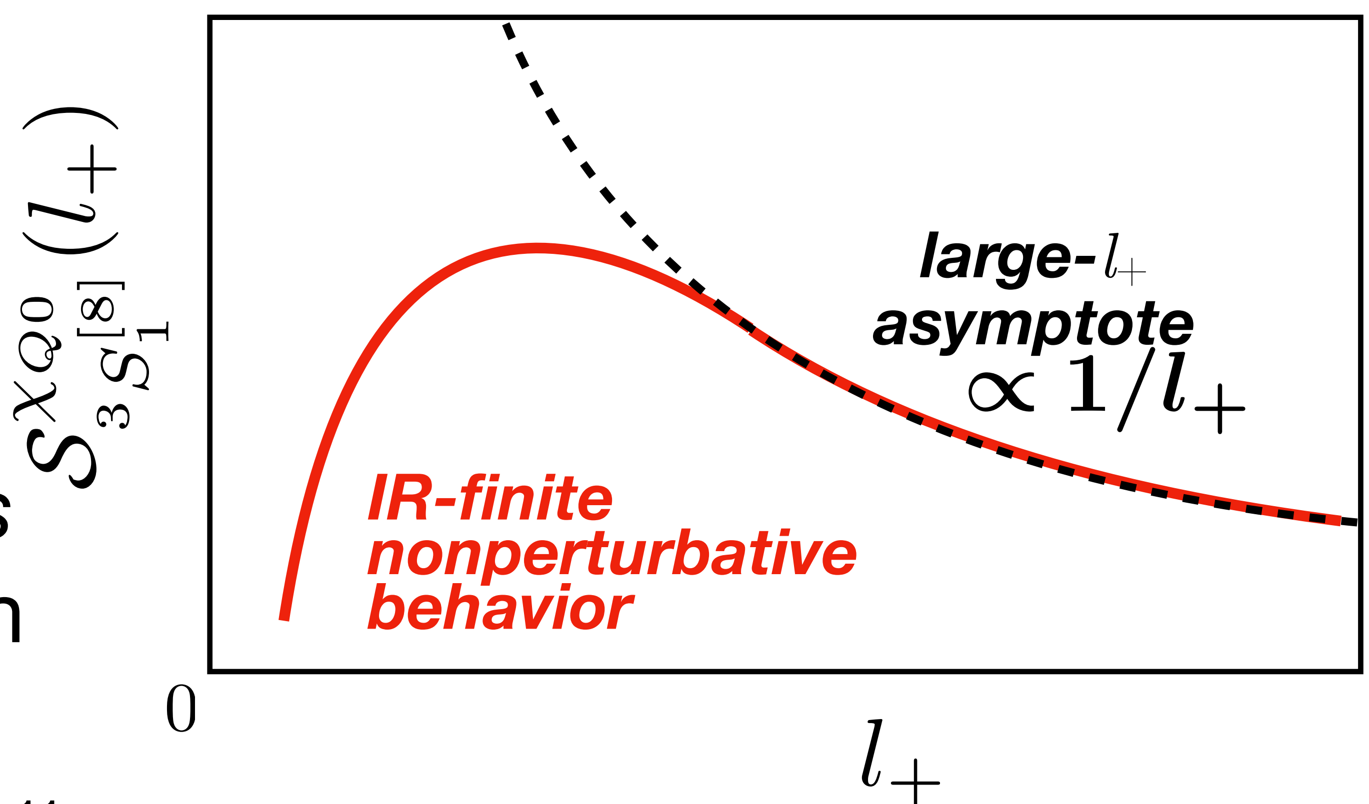
- *P*-wave short-distance coefficients have plus distributions with large subtractions, and color-singlet production rates are negative.

$$\int_0^{l_+^{\max}} \frac{dl_+}{l_+^{1+2\epsilon}} c_{^3S_1^{[8]}}(P+l) = -\frac{(l_+^{\max})^{-2\epsilon}}{2\epsilon} c_{^3S_1^{[8]}}(P) + \int_0^{l_+^{\max}} \frac{dl_+}{l_+} \left[c_{^3S_1^{[8]}}(P+l) - c_{^3S_1^{[8]}}(P) \right]$$

negative

- Nonperturbative shape function makes subtractions softer at $l_+ = 0$, color-singlet cross section is *less negative*

- *Hence, nonperturbative effects* enhance large-p_T cross section

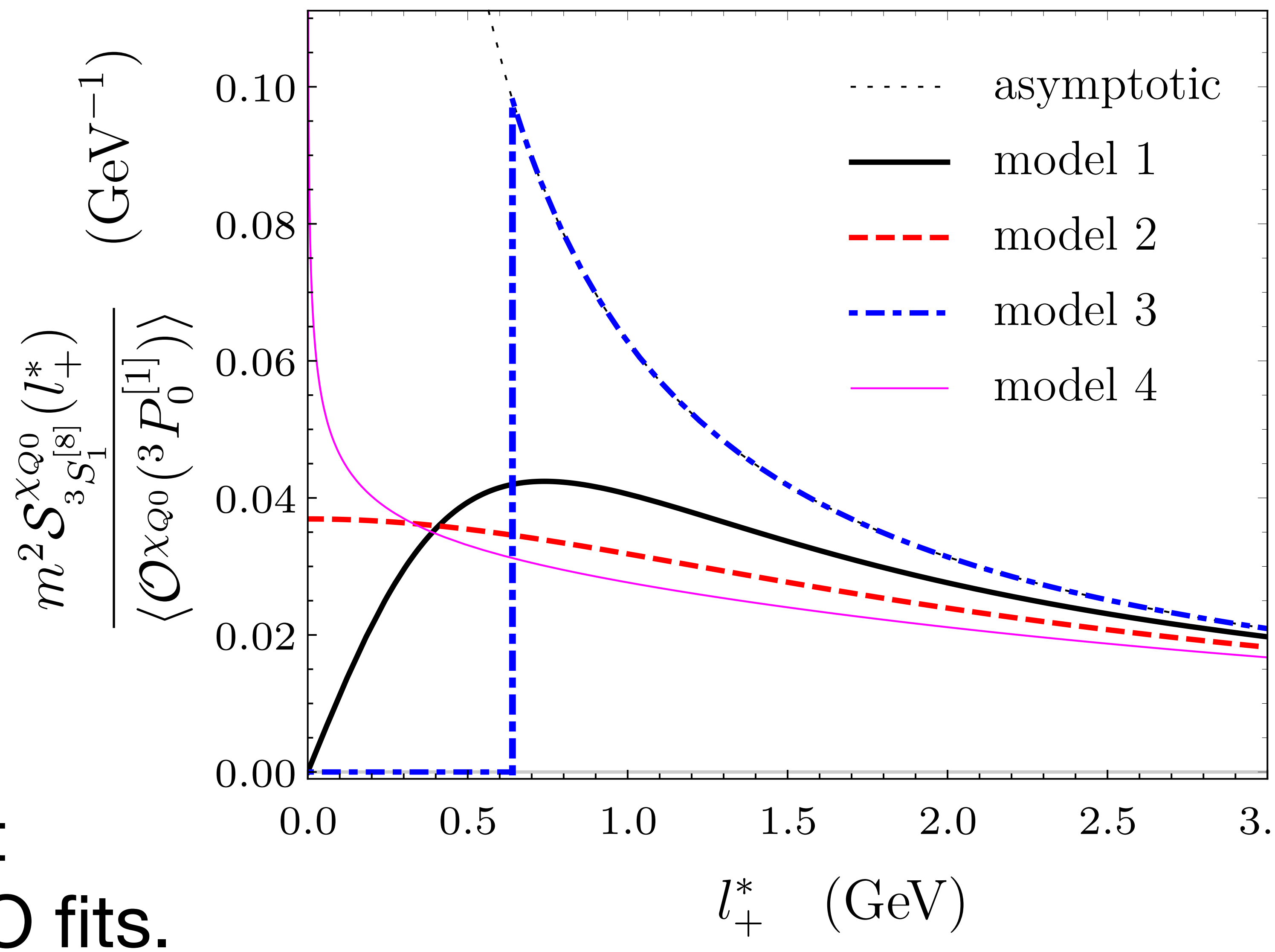


Models for Shape Function

- We can constrain models for the color-octet shape function using the asymptotic form and the normalization condition:

- $l_+ \to 0$ behavior is not strongly constrained, but it should not diverge like $1/l_+$ to ensure the IR-finiteness of the color-octet ME.

- All models shown here reproduce the color-octet matrix elements from NLO fits.

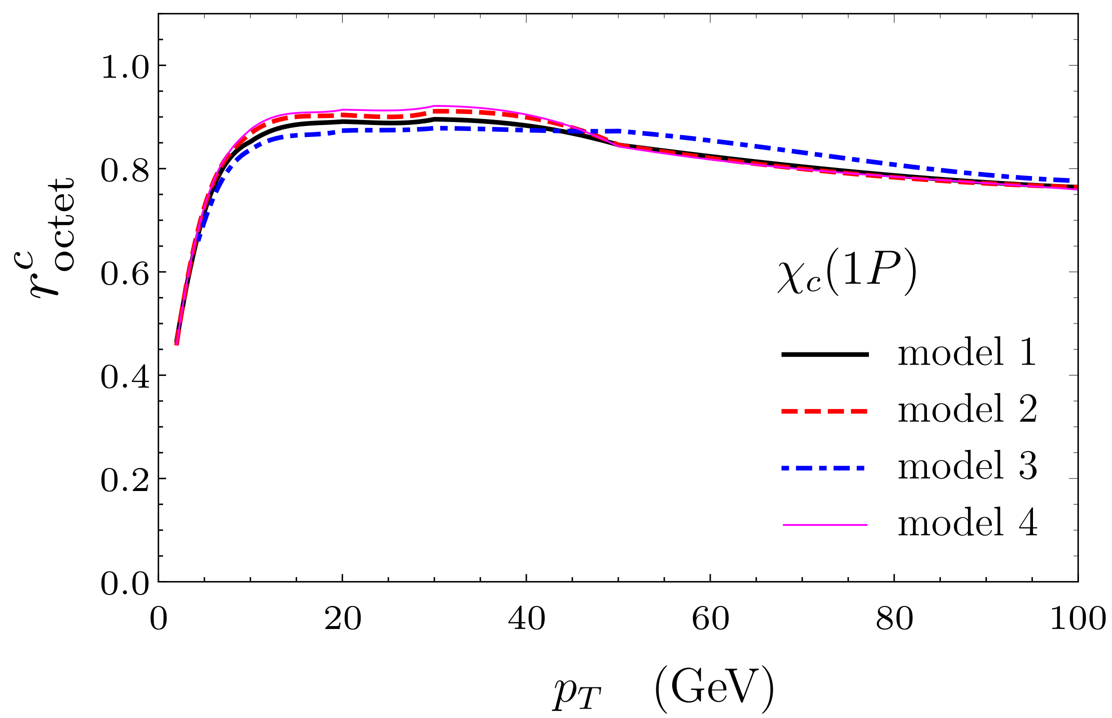


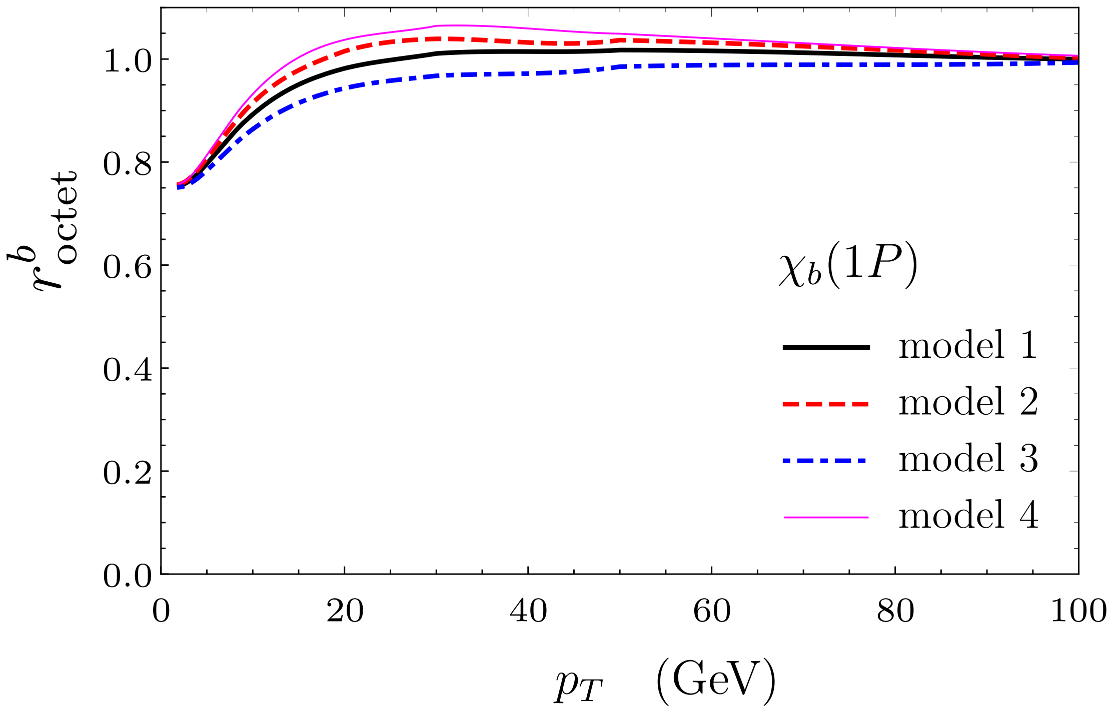
Corrections to Cross Section

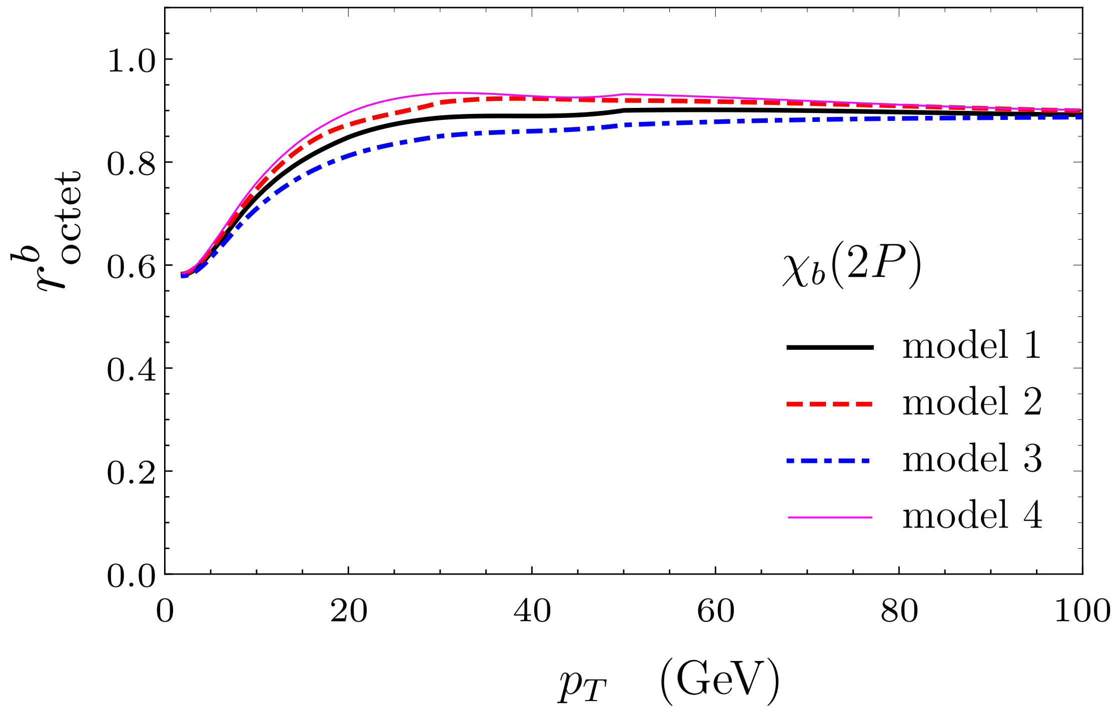
- Nonperturbative corrections are almost independent of model.

$$r = \frac{\text{octet cross section in shape function formalism}}{\text{octet cross section in NRQCD}}$$

- Ratio is constant at large p_T, and diminish as p_T decreases.
- Overall normalization decrease due to use of quarkonium mass instead of quark pole mass



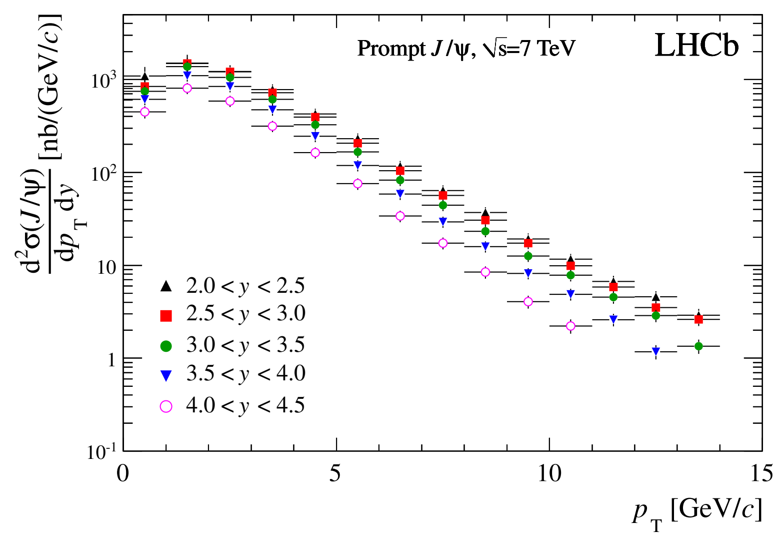




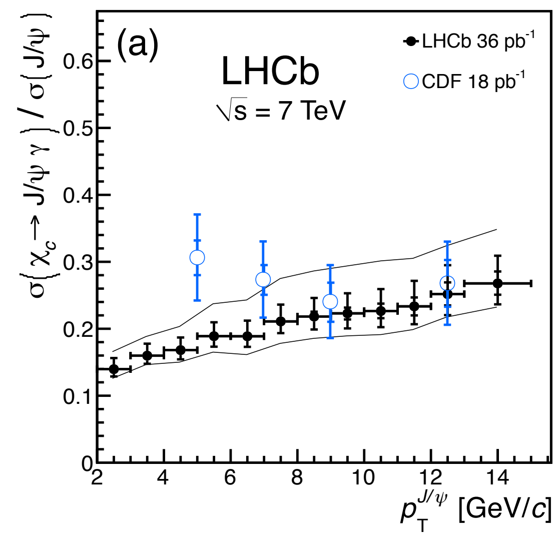
Cross Section Measurements

- Small-p_T data available from
  LHCb measurements of $\boldsymbol{\sigma}(J/\psi)$, $\boldsymbol{\sigma}(\psi(2S))$, $\boldsymbol{\sigma}(\Upsilon)$
  LHCb measurements of $\boldsymbol{\sigma}(\chi_c)/\boldsymbol{\sigma}(J/\psi)$, $\boldsymbol{\sigma}(\chi_b)/\boldsymbol{\sigma}(\Upsilon)$
  $\boldsymbol{\sigma}(\chi_c)$ and $\boldsymbol{\sigma}(\chi_b)$ can be obtained from their products.

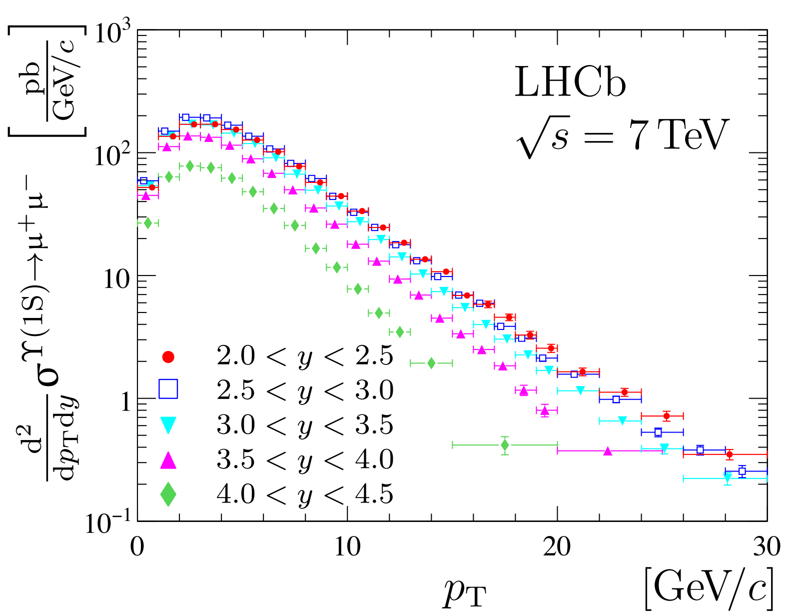
LHCb, EPJC71 (2011) 1645
PLB718 (2012) 431
EPJC74 (2014) 3092
JHEP11 (2015) 103



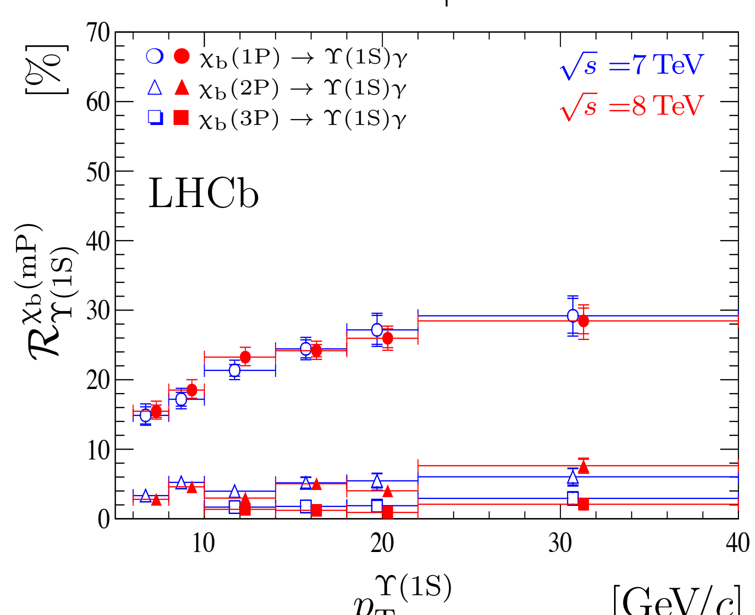
$\times$



$= \boldsymbol{\sigma}(\chi_c)$



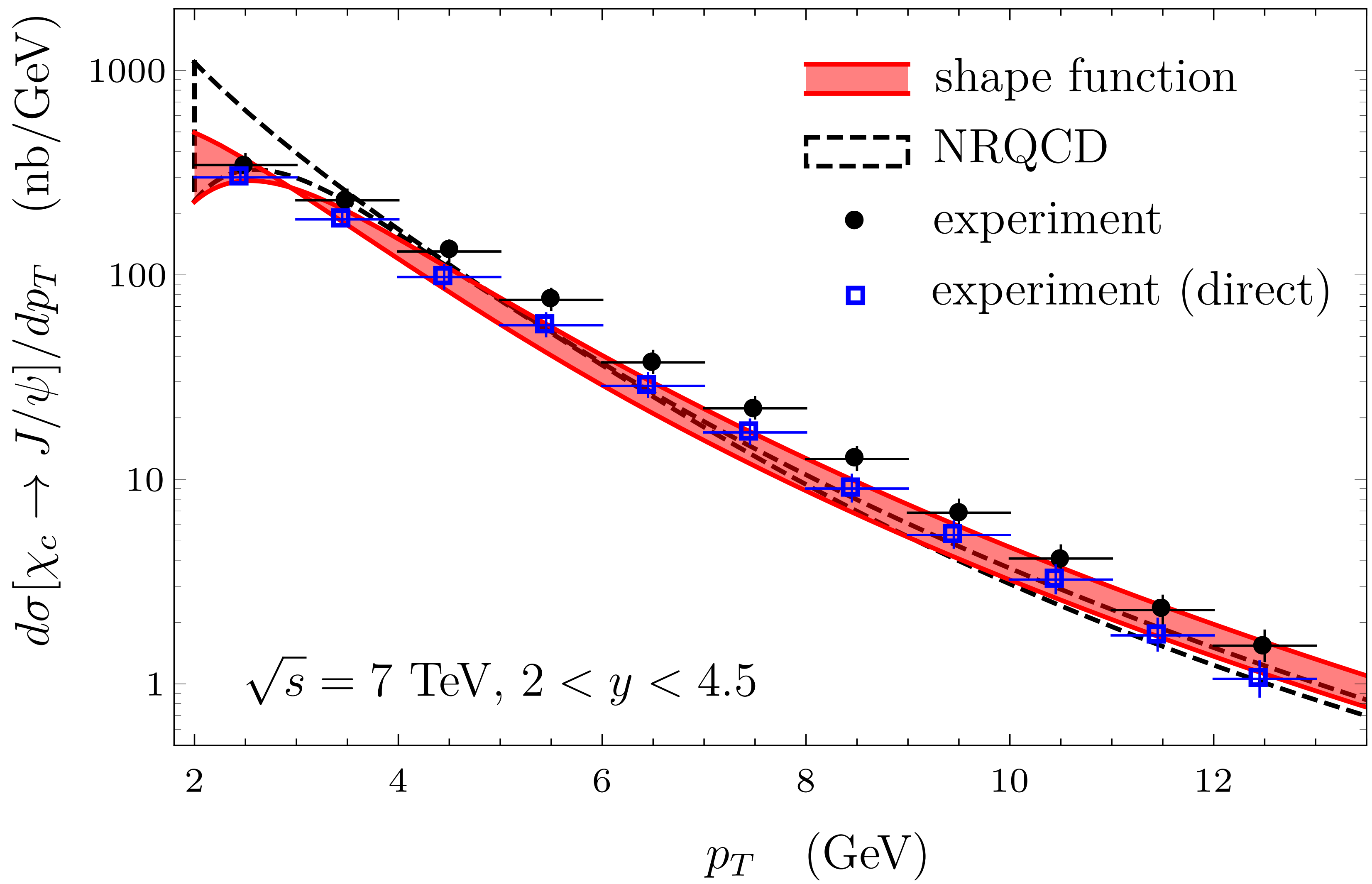
$\times$



$= \boldsymbol{\sigma}(\chi_b)$

χ_c Cross Sections

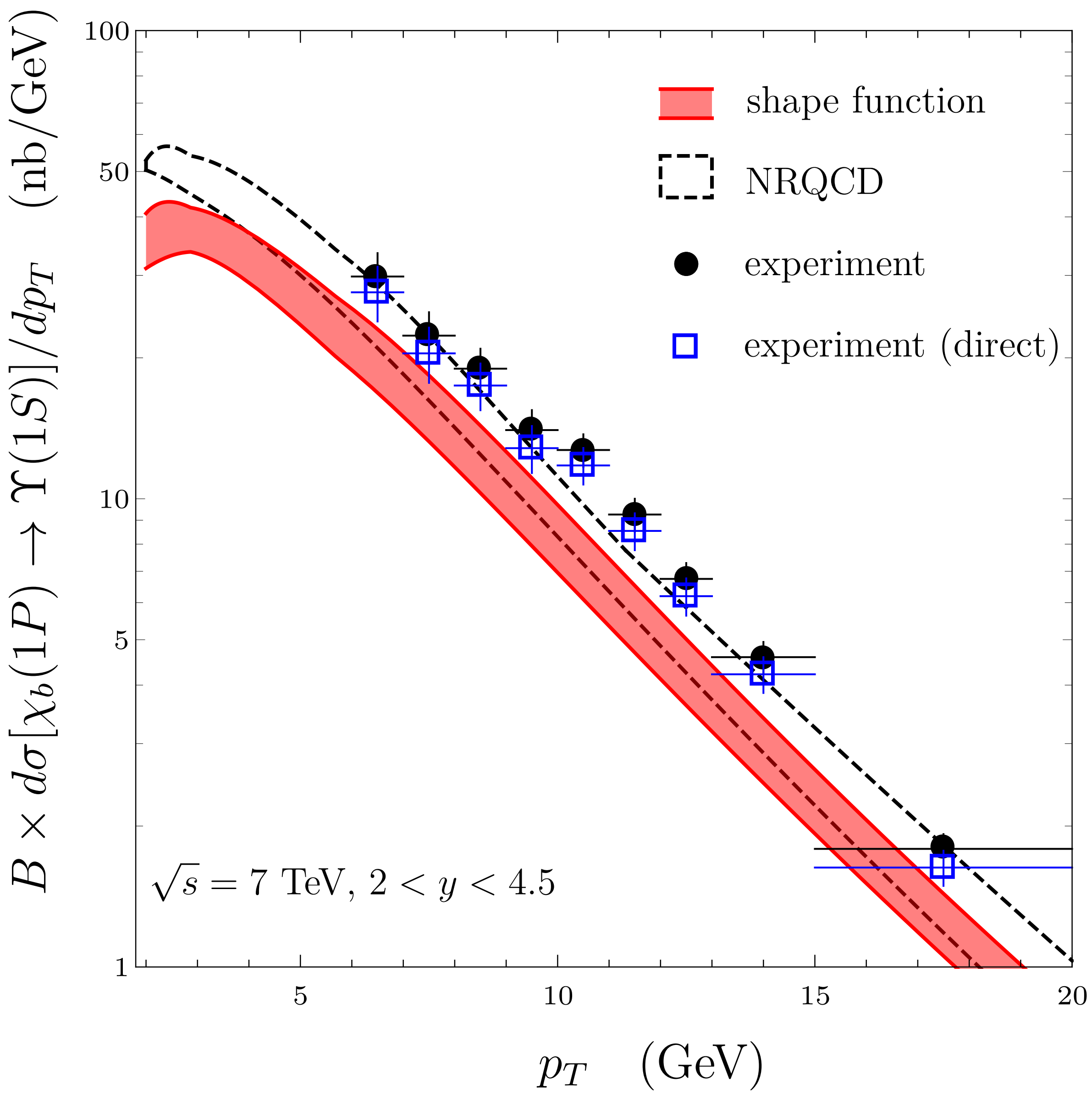
- *P*-wave charmonium cross sections

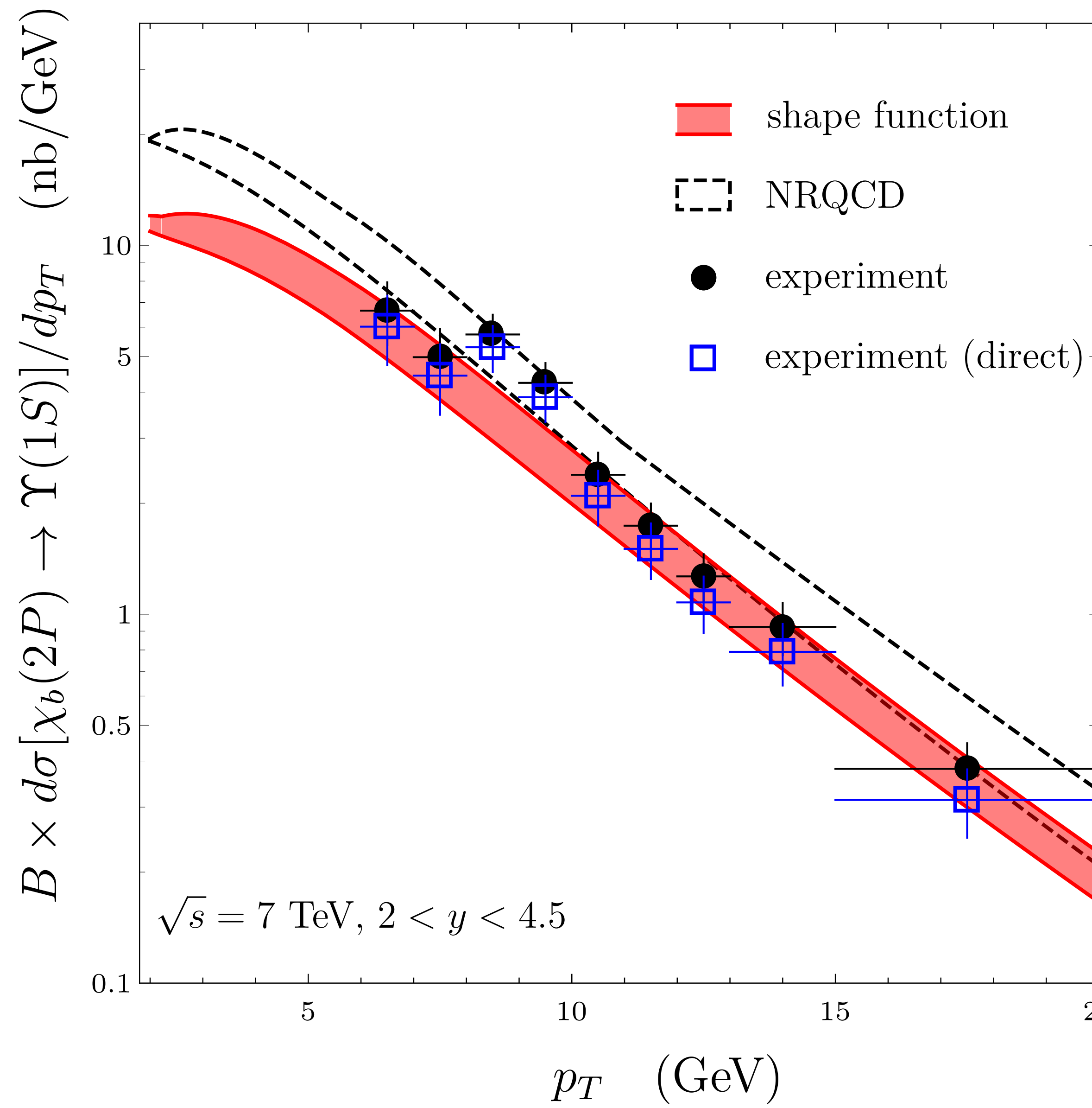


Experiment from LHCb, EPJC71 (2011) 1645, PLB718 (2012) 431 feeddown subtraction using LHCb, EPJC 72 (2012) 2100

χ_b Cross Sections

- *P*-wave bottomonium cross sections

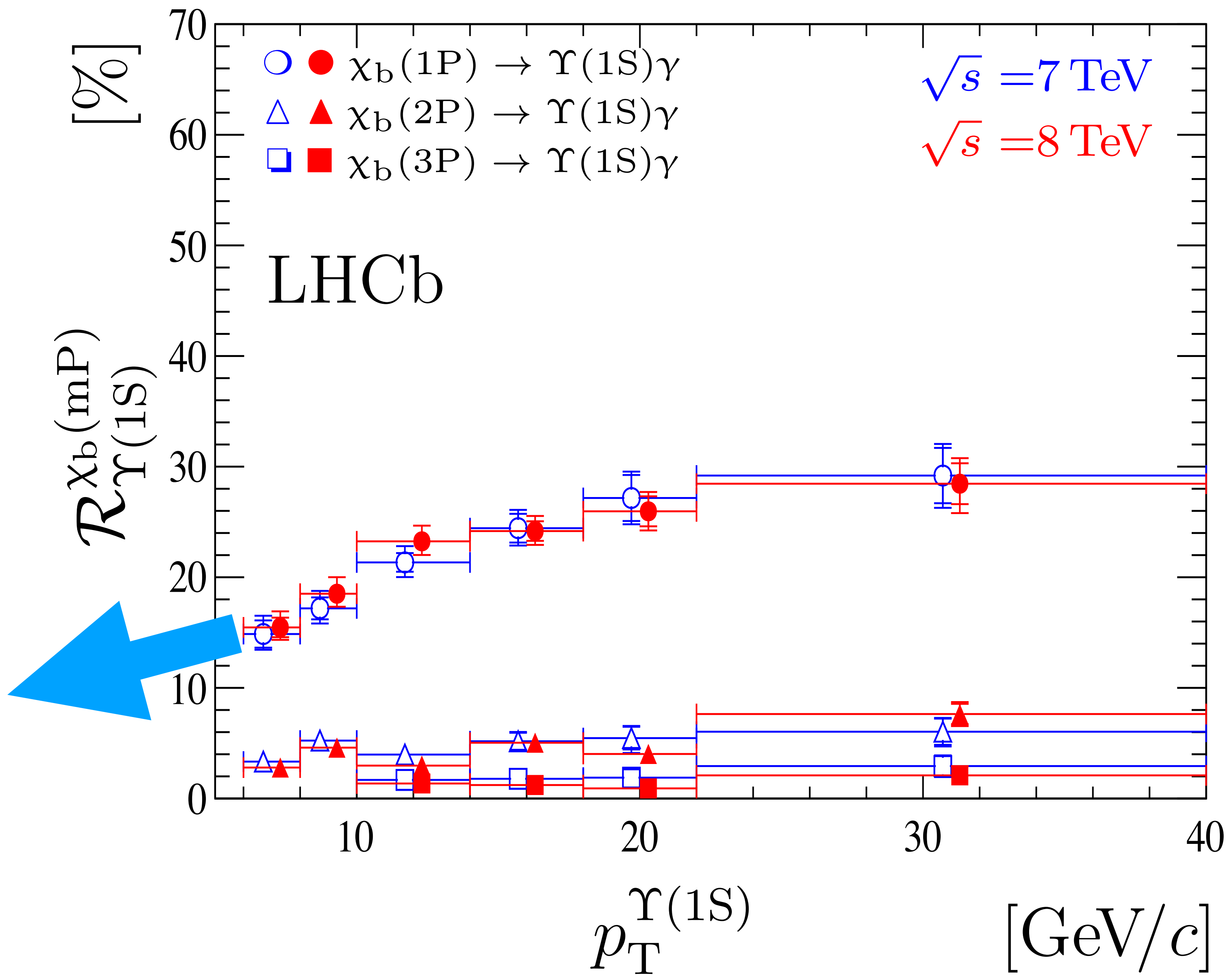




Experiment from LHCb, EPJC74 (2014) 3092, JHEP11 (2015) 103
feeddown subtraction using LHCb, JHEP11 (2015) 103

Going to lower p_T

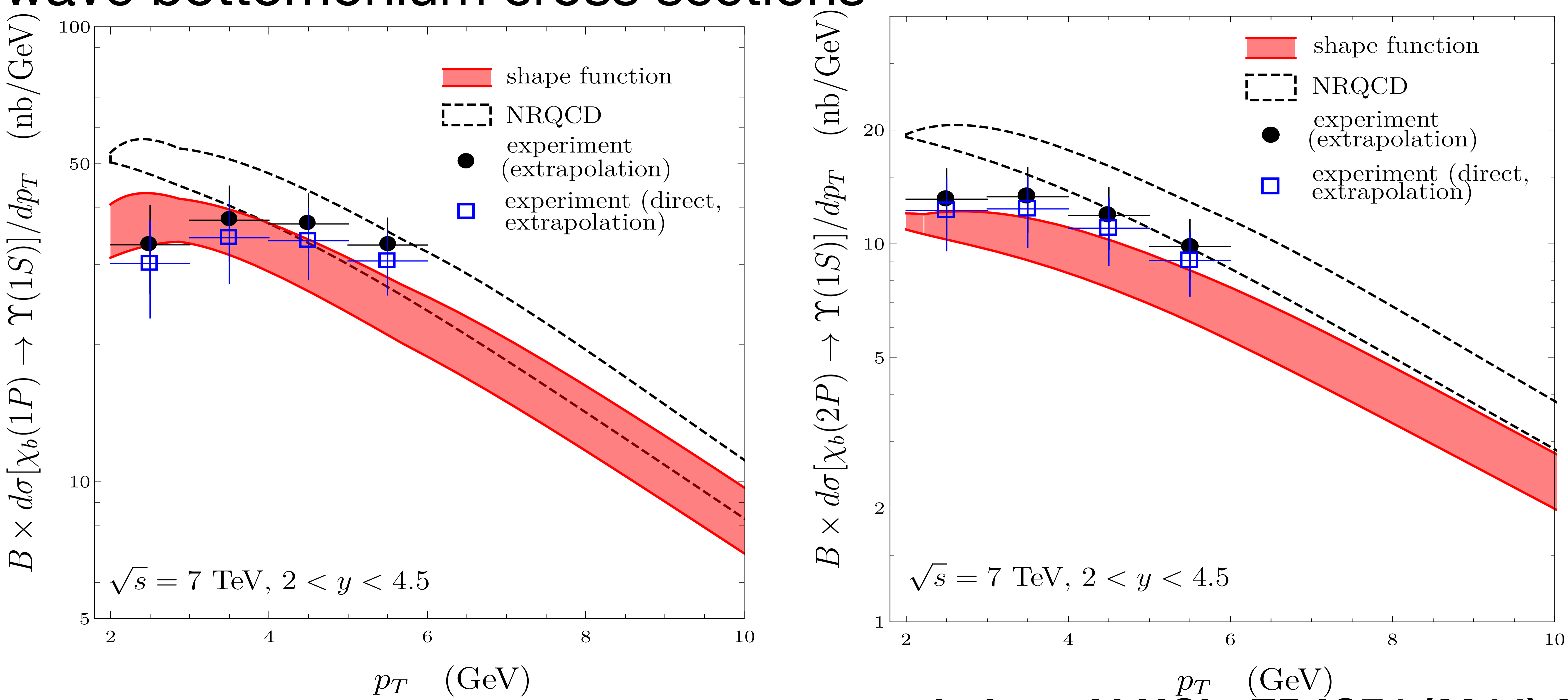
- If we were to extrapolate this down to even smaller p_T...



LHCb, [EPJC74 (2014) 3092]

χ_b Cross Sections

- P-wave bottomonium cross sections



extrapolation of LHCb, EPJC74 (2014) 3092

- A recent data-driven study suggests similar behaviors of small-p_T cross sections. **Boyd, Strickland, Thapa, arXiv:2307.03841 [hep-ph]**
- Although direct measurements are not available, knowledge of low-p_T behavior is important for treatment of feeddown in Υ production

Summary

- NRQCD involves *mixing* induced by *soft gluon emission*. Soft momentum can be important near boundaries of phase space.
- *Kinematical effects from soft momenta* can be ***resummed*** by shape function formalism, but this depends on unknown nonperturbative functions. Phenomenological application was *very limited*.
- This work revealed relation between ***shape function formalism*** and ***renormalization in NRQCD***. This severely constrains model dependence and ***restores predictability*** to standard NRQCD level.
- Inclusion of nonperturbative kinematical corrections ***soften the small-p_T behavior*** of χ_c, χ_b cross sections, potentially improving theory description of p_T-dependent χ_c, χ_b cross sections.
- Similar mixing happen in J/ψ, $\psi(2S)$, Υ production: application of shape function formalism to S-wave production can be anticipated