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CHARM 2023
Siegen, Germany
Hörsaalzentrum am Unteren Schloss



Doubly Charmed Baryon Decays in Quark Models

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July 20, 2023



In collab with Hai-Yang Cheng, Peng-Yu Niu, Shuge Zeng



OUTLINE

- Introduction
- Model setup
- Non-perturbative parameters in quark models
- Results and discussion

INTRODUCTION

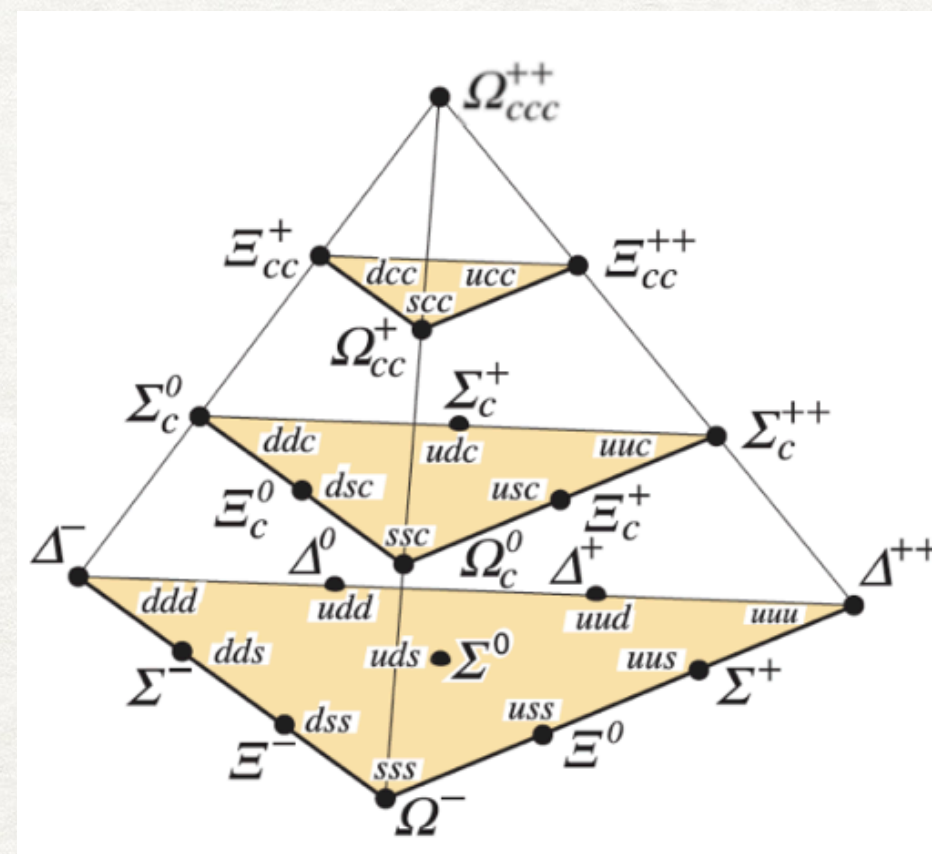
Towards charmed baryons

- Charm quark is special for its mass implying the theoretical challenge.
talk given by K. Vos
- Baryons provide a good platform to study charm physics.

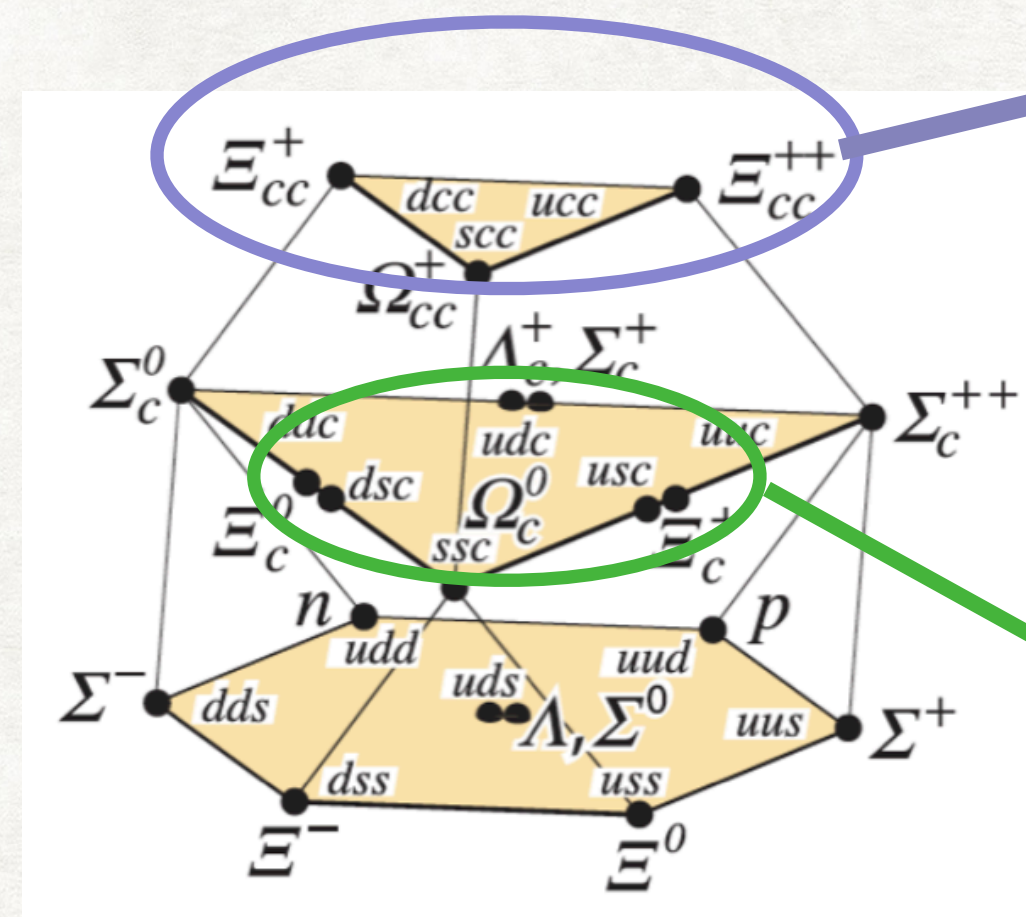
Kräuter Quark
Herb quark



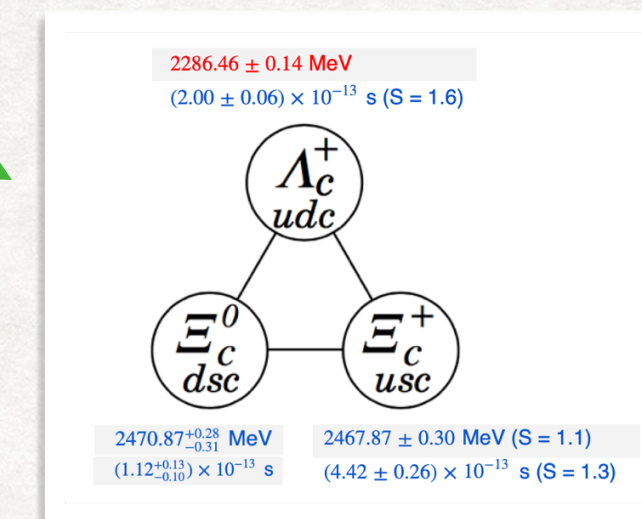
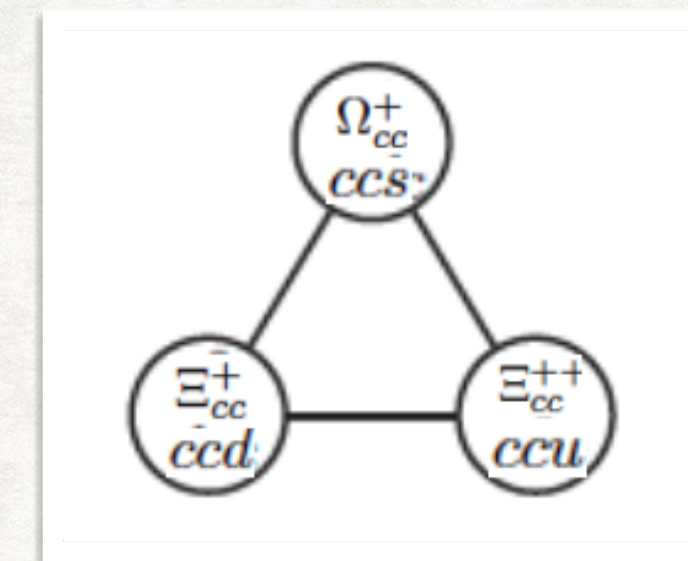
Siegen breakfast



SU(4) 20-plet with an SU(3) decuplet



SU(4) 20-plet with an SU(3) octet



- Topics of charmed baryons: **lifetimes**, **masses**, **decays**, CPV...

H.-Y. Cheng, B. Melic, A. Schwartz ...

S. Collins, P. Spradlin, ...

H.-Y. Cheng, P. Spradlin, Y. Xu...
& this talk

Charm 2025 ?

INTRODUCTION

Doubly charmed baryon family

- The first observed doubly charmed baryon Ξ_{cc}^{++} : mass and lifetime

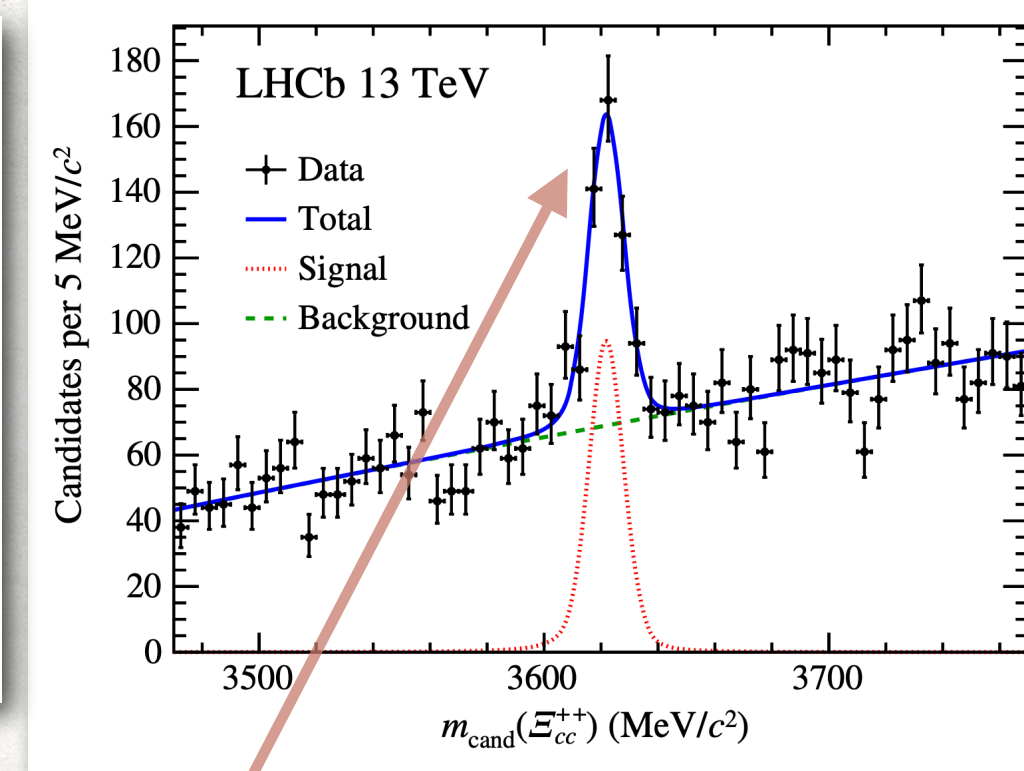
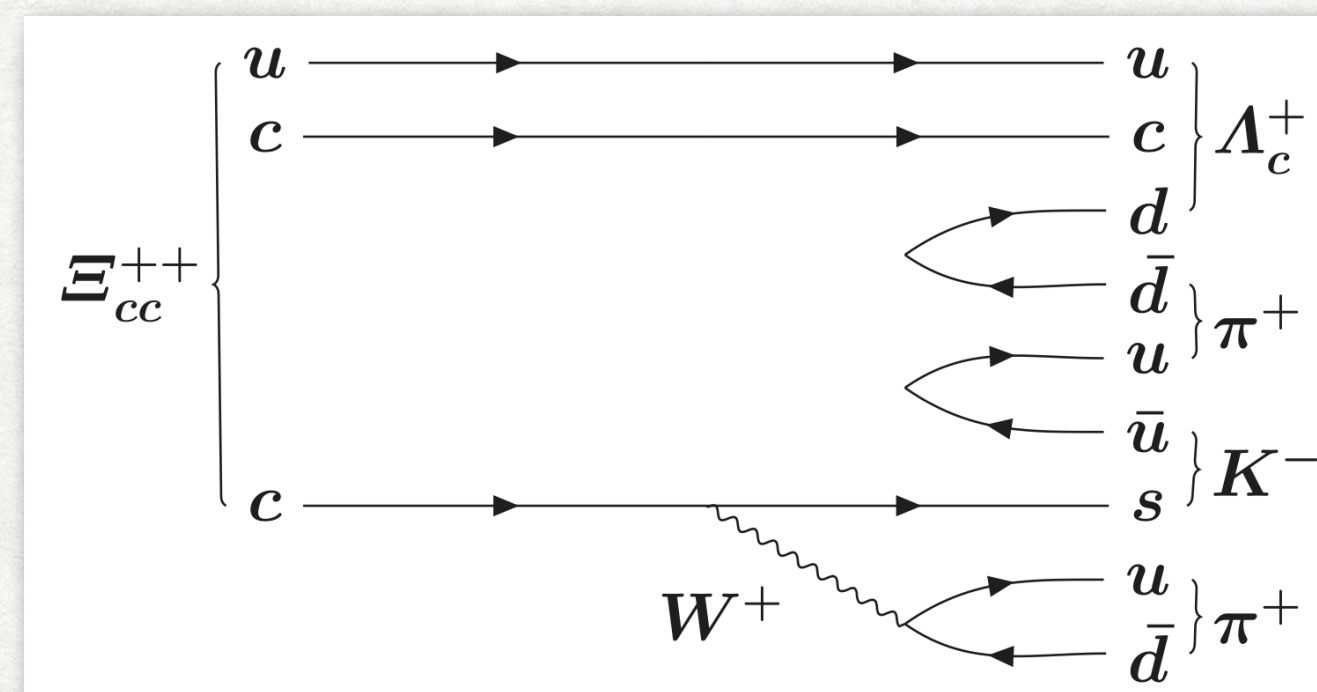
PRL 119, 112001 (2017) week ending
15 SEPTEMBER 2017

Selected for a **Viewpoint** in *Physics*
PHYSICAL REVIEW LETTERS

Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.**
(LHCb Collaboration)

(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)



$$3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+) \text{ MeV}/c^2$$

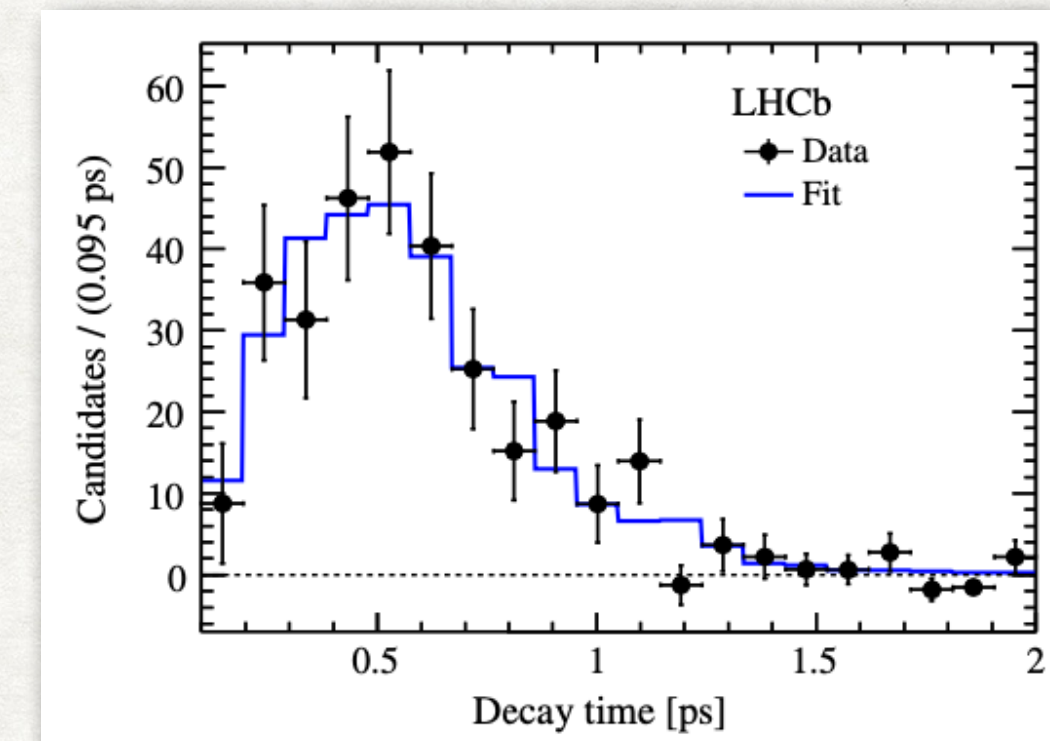
PHYSICAL REVIEW LETTERS 121, 052002 (2018)

Editors' Suggestion

Measurement of the Lifetime of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.**
(LHCb Collaboration)

(Received 7 June 2018; revised manuscript received 24 June 2018; published 31 July 2018)



$$\tau(\Xi_{cc}^{++}) = 0.256_{-0.022}^{+0.024}(\text{stat}) \pm 0.014(\text{syst}) \text{ ps}$$

INTRODUCTION

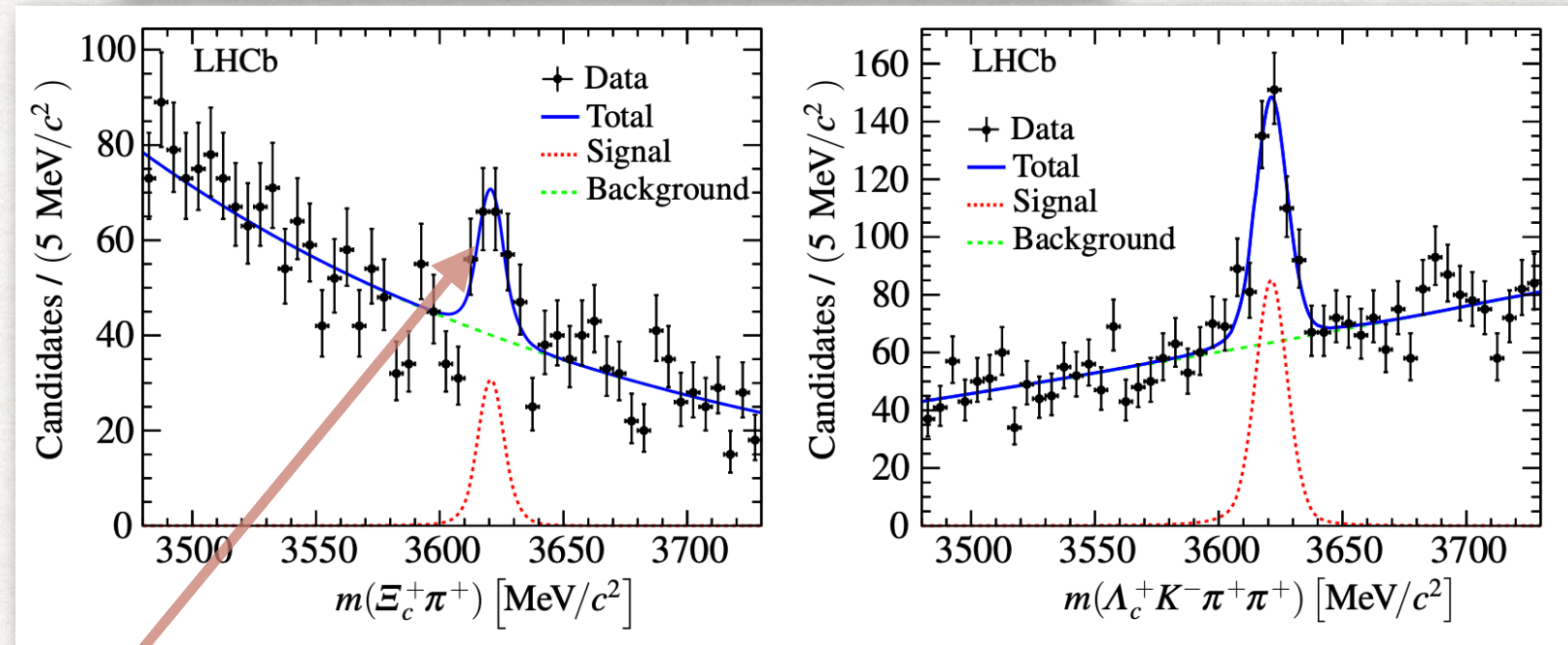
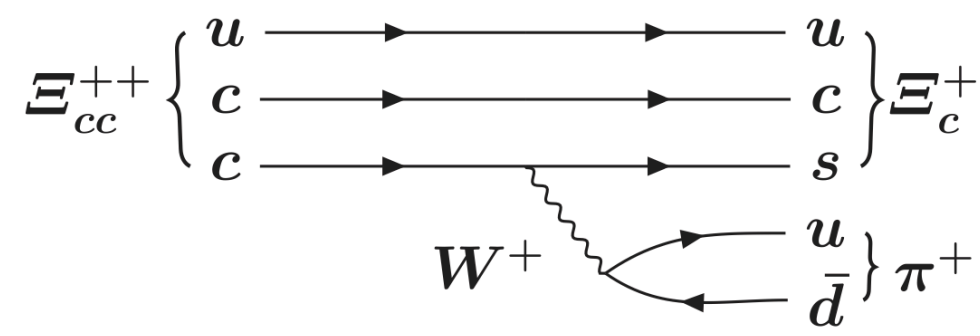
Decays of Ξ_{cc}^{++}

PHYSICAL REVIEW LETTERS **121**, 162002 (2018)

First Observation of the Doubly Charmed Baryon Decay $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$

R. Aaij *et al.**
(LHCb Collaboration)

(Received 9 July 2018; revised manuscript received 1 August 2018; published 17 October 2018)



$$3620.6 \pm 1.5(\text{stat}) \pm 0.4(\text{syst}) \pm 0.3(\Xi_c^+) \text{ MeV}/c^2$$

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) \times \mathcal{B}(\Xi_c^+ \rightarrow p K^- \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+) \times \mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)} = 0.035 \pm 0.009(\text{stat}) \pm 0.003(\text{syst}).$$



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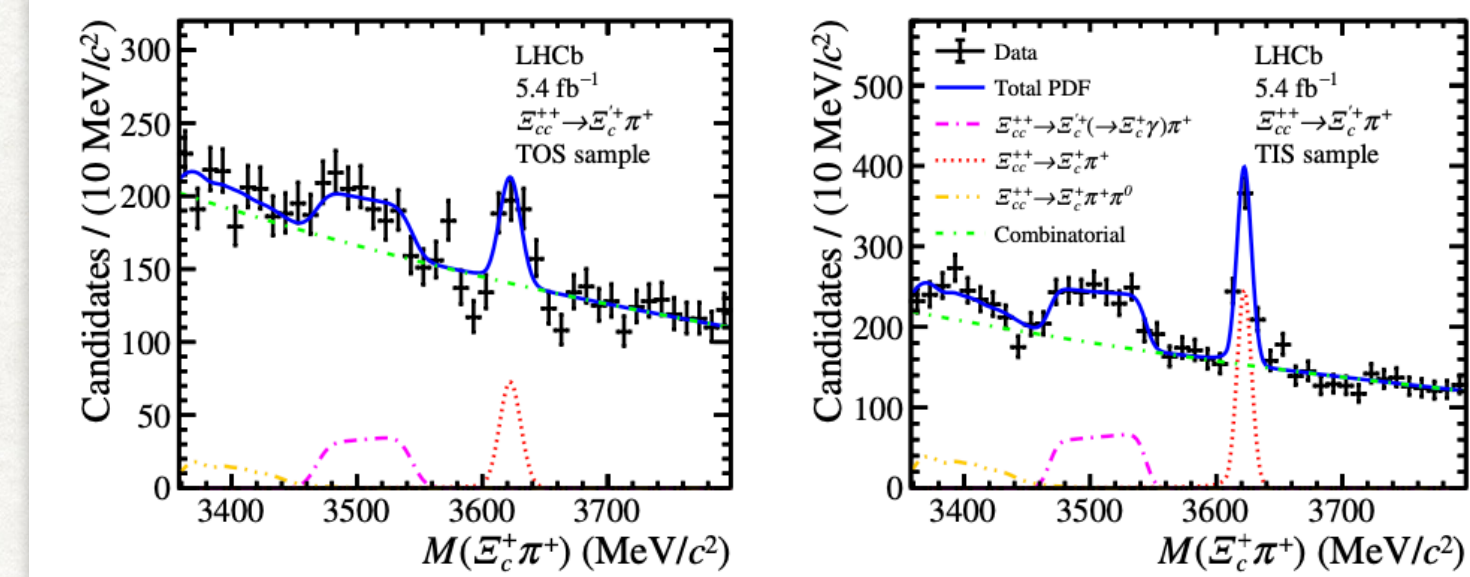
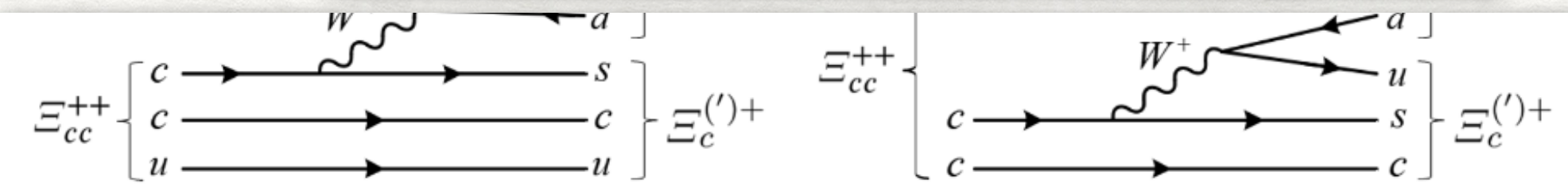
PUBLISHED: May 6, 2022

Observation of the doubly charmed baryon decay

$$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$$



The LHCb collaboration



$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)} = 1.41 \pm 0.17 \pm 0.10$$

INTRODUCTION

Extracting information of Ξ_{cc}^{++} decays from experimental results

$$R \equiv \frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)} = 1.41 \pm 0.17 \pm 0.10$$

LHCb JHEP 05 (2022), 038

$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) \times \mathcal{B}(\Xi_c^+ \rightarrow p K^- \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+) \times \mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+)} = 0.035 \pm 0.009(\text{stat.}) \pm 0.003(\text{syst.})$$

LHCb PRL 121,162002 (2018)

$$\mathcal{B}(\Lambda_c^+ \rightarrow p K^- \pi^+) = (6.28 \pm 0.32)\%$$

$$\mathcal{B}(\Xi_c^+ \rightarrow p K^- \pi^+) = (0.62 \pm 0.30)\%$$

PDG



$$\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+)} = 0.35 \pm 0.20$$



$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+) \approx \frac{2}{3} \mathcal{B}(\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \bar{K}^{*0})$$

$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Sigma_c^{++} \bar{K}^{*0}) = 5.61\%$$

T. Gutsche et al., PRD 100, 114037 (2019)

$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)_{\text{expt}} \approx (1.33 \pm 0.74)\%$$

INTRODUCTION

- Incomplete list of recent papers on Ξ_{cc}^{++} decays

- [1] W. Wang, Z. P. Xing and J. Xu, “Weak Decays of Doubly Heavy Baryons: SU(3) Analysis,” Eur. Phys. J. C **77**, 800 (2017) [arXiv:1707.06570 [hep-ph]].
- [2] A. S. Gerasimov and A. V. Luchinsky, “Weak decays of doubly heavy baryons: Decays to a system of π mesons,” Phys. Rev. D **100**, 073015 (2019) [arXiv:1905.11740 [hep-ph]].
- [3] H. W. Ke, F. Lu, X. H. Liu and X. Q. Li, “Study on $\Xi_{cc} \rightarrow \Xi_c$ and $\Xi_{cc} \rightarrow \Xi'_c$ weak decays in the light-front quark model,” Eur. Phys. J. C **80**, no.2, 140 (2020) [arXiv:1912.01435 [hep-ph]].
- [4] Y. J. Shi, W. Wang and Z. X. Zhao, “QCD Sum Rules Analysis of Weak Decays of Doubly-Heavy Baryons,” Eur. Phys. J. C **80**, no.6, 568 (2020) [arXiv:1902.01092 [hep-ph]].
- [5] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and Z. Tyulemissov, “Ab initio three-loop calculation of the W -exchange contribution to nonleptonic decays of double charm baryons,” Phys. Rev. D **99**, 056013 (2019) [arXiv:1812.09212 [hep-ph]].
- [6] Y. J. Shi, Z. X. Zhao, Y. Xing and U. G. Meißner, “ W -exchange contribution to the decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ using light-cone sum rules,” Phys. Rev. D **106** (2022) no.3, 034004 [arXiv:2206.13196 [hep-ph]].
- [7] J. J. Han, H. Y. Jiang, W. Liu, Z. J. Xiao and F. S. Yu, “Rescattering mechanism of weak decays of double-charm baryons,” Chin. Phys. C **45**, no.5, 053105 (2021) [arXiv:2101.12019 [hep-ph]].
- [8] N. Sharma and R. Dhir, “Estimates of W -exchange contributions to Ξ_{cc} decays,” Phys. Rev. D **96** (2017) no.11, 113006 [arXiv:1709.08217 [hep-ph]].
- [9] H. W. Ke and X. Q. Li, “Revisiting the transition $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+}$ to understand the data from LHCb,” Phys. Rev. D **105** (2022) no.9, 096011 [arXiv:2203.10352 [hep-ph]].

- [11] H. Y. Cheng, G. Meng, F. Xu and J. Zou, Phys. Rev. D **101**, no.3, 034034 (2020) doi:10.1103/PhysRevD.101.034034 [arXiv:2001.04553 [hep-ph]].
- [12] S. Zeng, F. Xu, P. Y. Niu and H. Y. Cheng, Phys. Rev. D **107**, no.3, 034009 (2023) doi:10.1103/PhysRevD.107.034009 [arXiv:2212.12983 [hep-ph]].

KINEMATICS

- Partial wave amplitudes

$$M(\mathcal{B}_i \rightarrow \mathcal{B}_f P) = i\bar{u}_f(A - B\gamma_5)u_i,$$

dynamics

- Physical observables

$$\Gamma = \frac{p_c}{8\pi} \left[\frac{(m_i + m_f)^2 - m_P^2}{m_i^2} |A|^2 + \frac{(m_i - m_f)^2 - m_P^2}{m_i^2} |B|^2 \right]$$

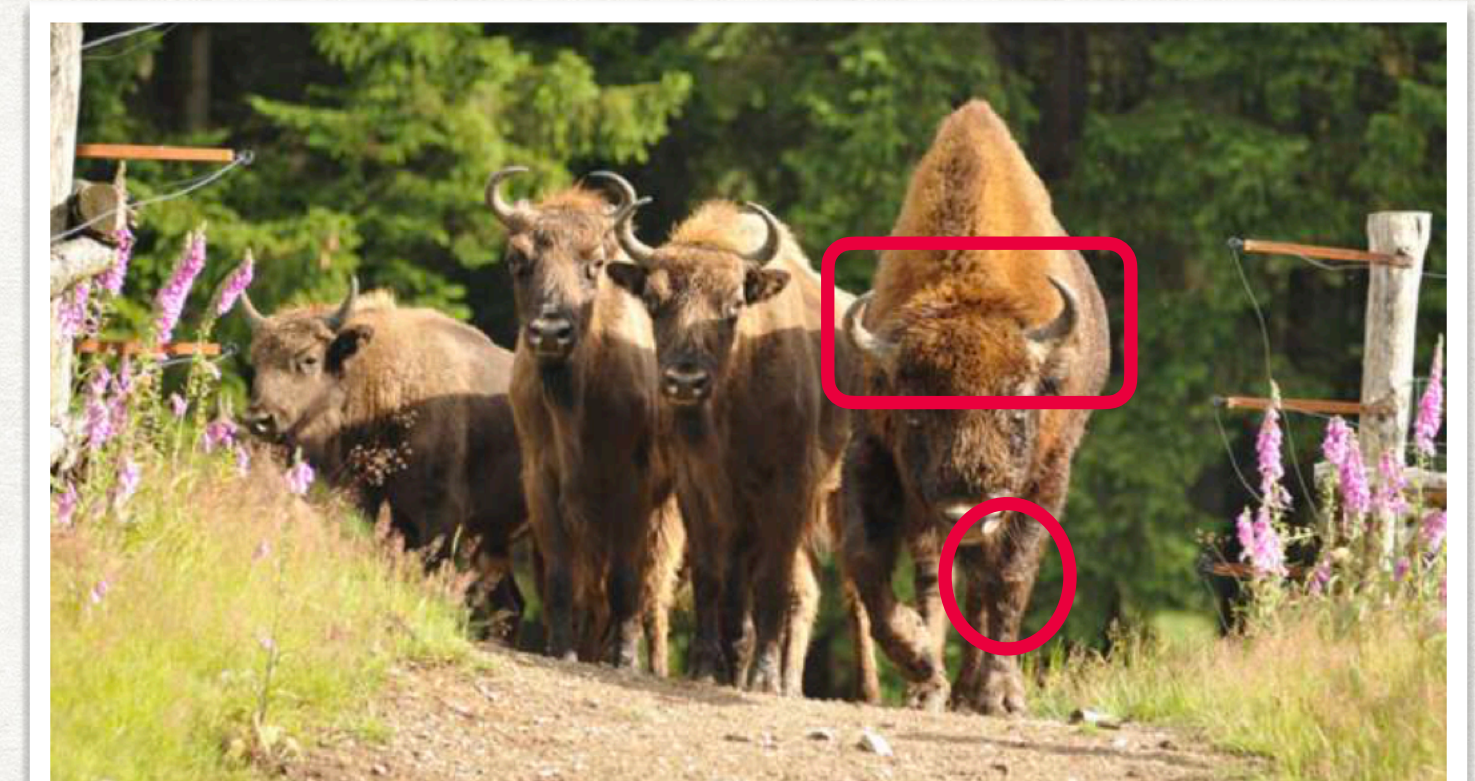
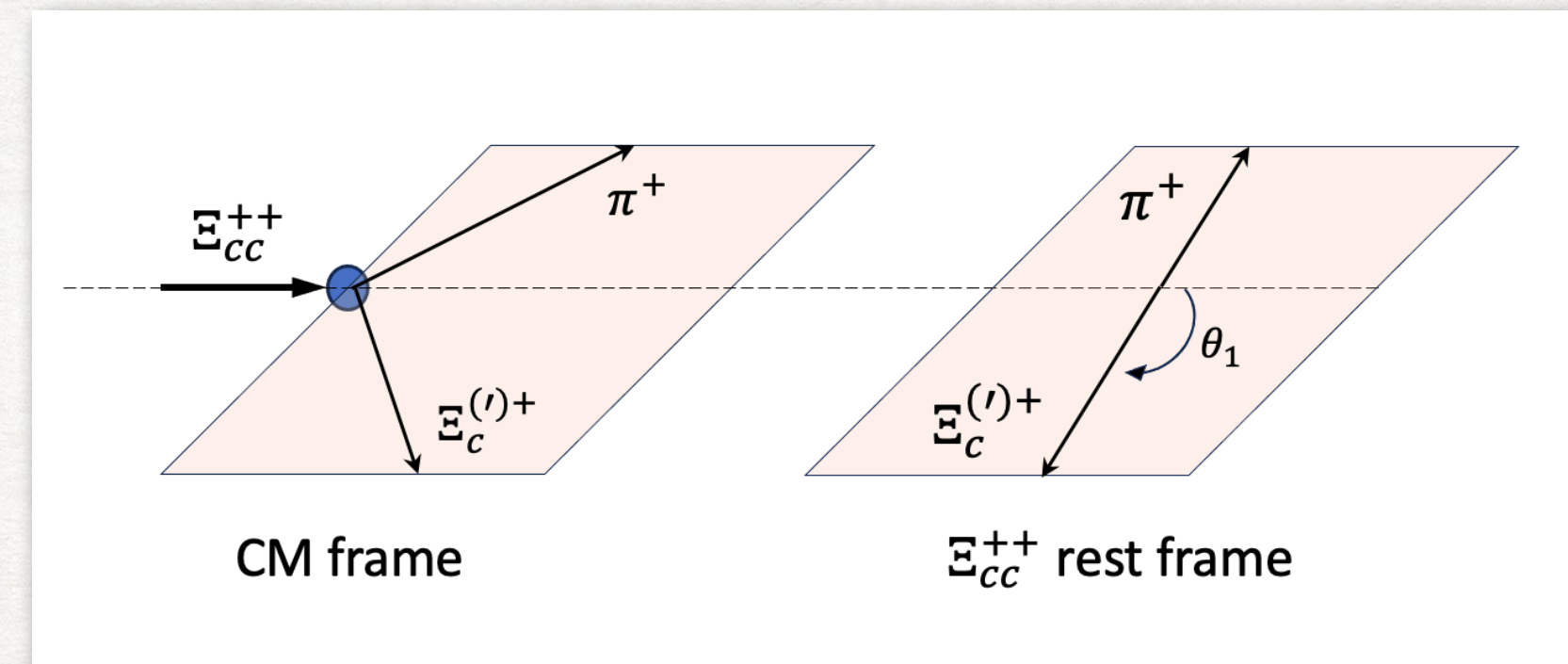
decay width or
branching fraction

absolute size of amplitudes

$$\alpha = \frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2 |B|^2}$$

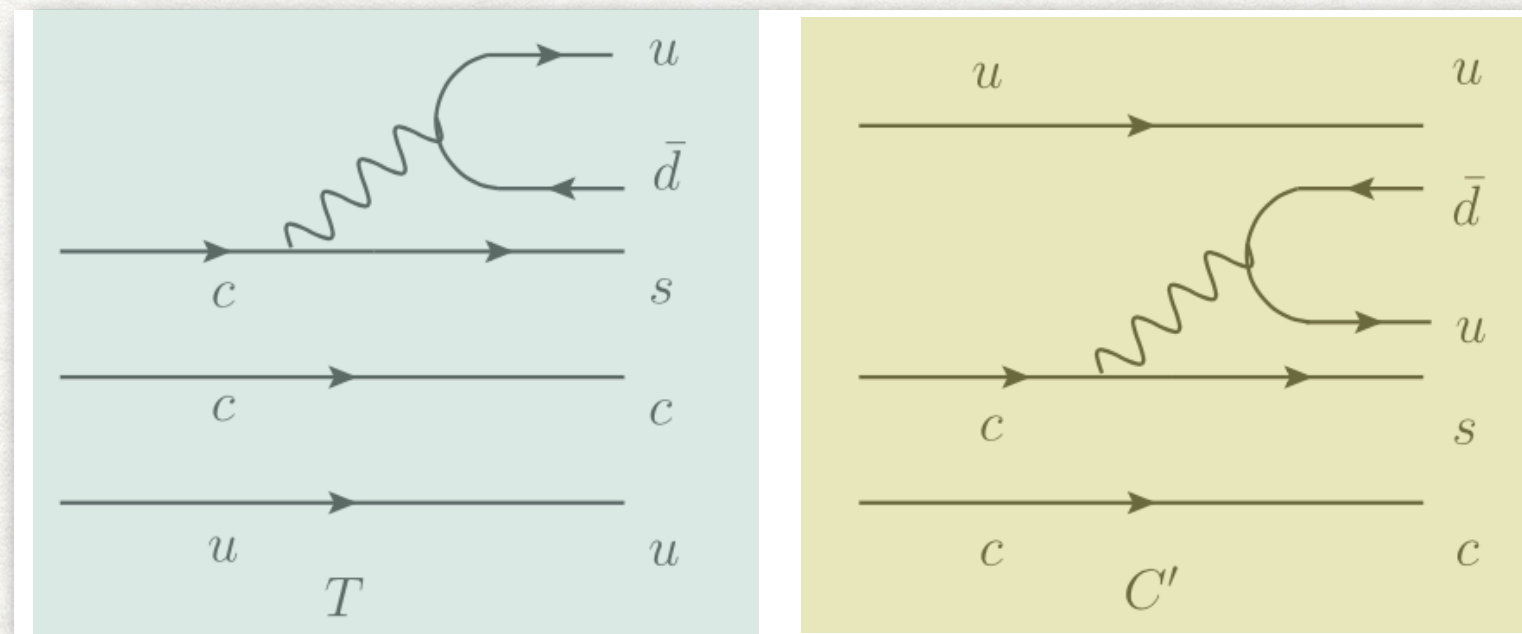
longitudinal polarization

relative size and phase of amplitudes



DYNAMICS: TOPOLOGICAL DIAGRAMS & POLE MODEL

- topological diagram method
 - a tool to manipulate fit (meson case: Chau & Cheng PRL56(1986)1655)
 - **a tool to assist in analyzing dynamics**

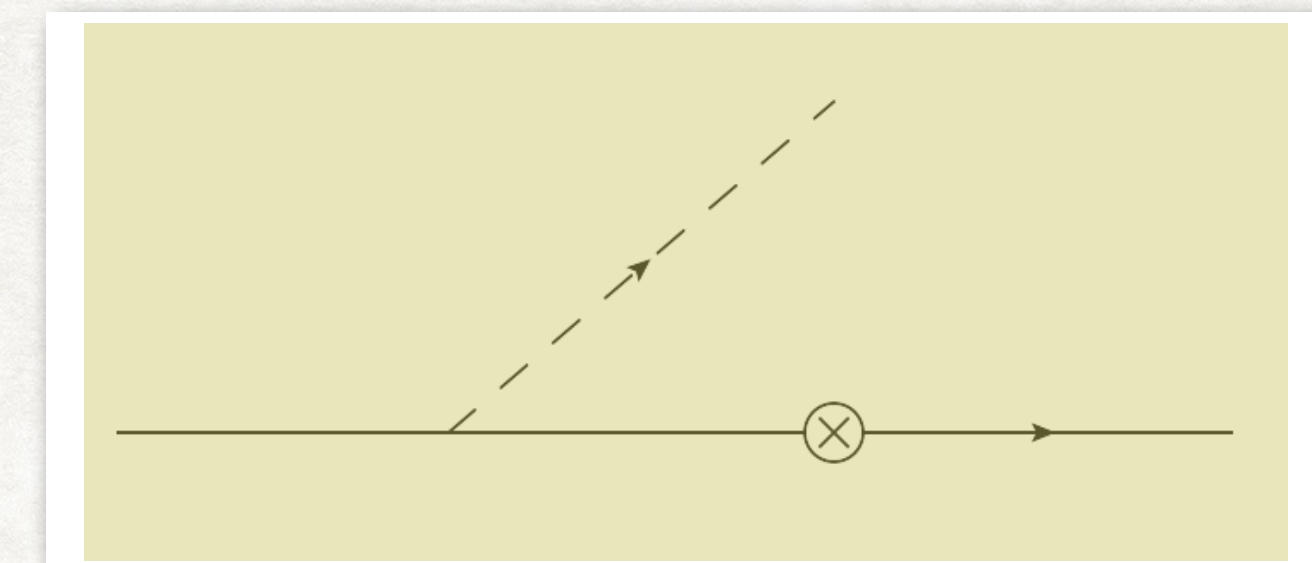
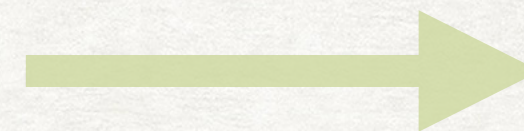


factorizable

non-factorizable

$$A^{\text{fac}} = \frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{B_{cc}} - m_{B_c}) f_1(q^2),$$

$$B^{\text{fac}} = -\frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{B_{cc}} + m_{B_c}) g_1(q^2)$$



$$A^{\text{pole}} = - \sum_{B_n^*(1/2^-)} \left[\frac{g_{B_f B_n^* P} b_{n^* i}}{m_i - m_{n^*}} + \frac{b_{f n^*} g_{B_n^* B_i P}}{m_f - m_{n^*}} \right],$$

$$B^{\text{pole}} = \sum_{B_n} \left[\frac{g_{B_f B_n P} a_{ni}}{m_i - m_n} + \frac{a_{fn} g_{B_n B_i P}}{m_f - m_n} \right],$$

DETAILED AMPLITUDES

A summary of all amplitudes

factorizable
amplitudes

$$A^{\text{fac}} = \frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{B_{cc}} - m_{B_c}) f_1(q^2),$$

$$B^{\text{fac}} = -\frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{B_{cc}} + m_{B_c}) g_1(q^2)$$

non-perturbative parameters

non-factorizable
amplitudes

$$A^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) = \frac{1}{f_\pi} (-a_{\Xi_c^+ \Xi_{cc}^+}),$$

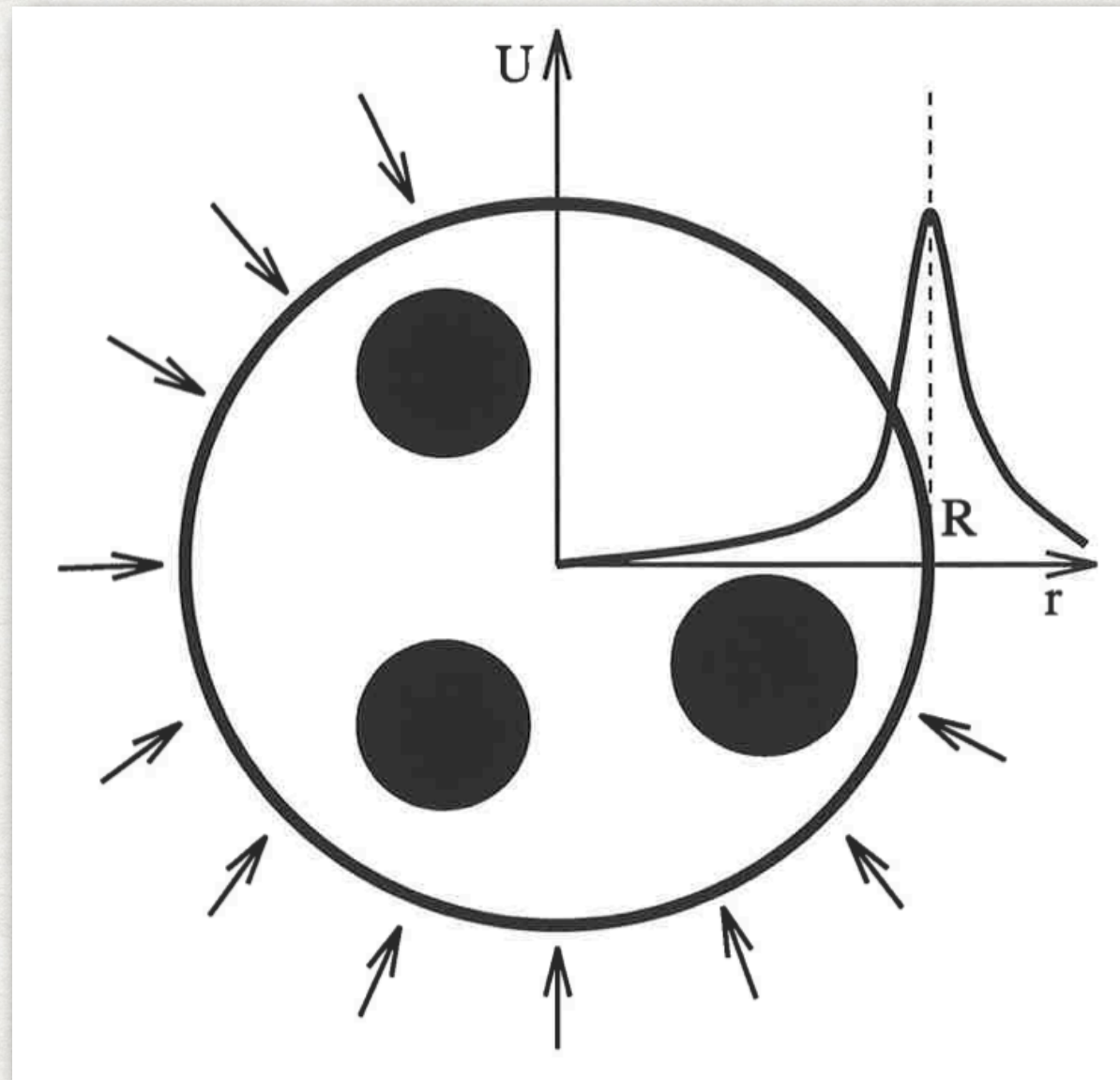
$$A^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+) = \frac{1}{f_\pi} (-a_{\Xi_c'^+ \Xi_{cc}^+}),$$

$$B^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+) = \frac{1}{f_\pi} \left(a_{\Xi_c^+ \Xi_{cc}^+} \frac{m_{\Xi_{cc}^{++}} + m_{\Xi_c^+}}{m_{\Xi_c^+} - m_{\Xi_{cc}^+}} g_{\Xi_{cc}^+ \Xi_{cc}^{++}}^{A(\pi^+)} \right),$$

$$B^{\text{nf}}(\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+) = \frac{1}{f_\pi} \left(a_{\Xi_c'^+ \Xi_{cc}^+} \frac{m_{\Xi_{cc}^{++}} + m_{\Xi_c'^+}}{m_{\Xi_c'^+} - m_{\Xi_{cc}^+}} g_{\Xi_{cc}^+ \Xi_{cc}^{++}}^{A(\pi^+)} \right),$$

- Lattice QCD
- QCD sum rule
- **Quark Models**

MIT BAG MODEL (STATIC BAG MODEL)



MIT bag model

$$[i\gamma^\mu \partial_\mu - (m + U)] \psi = 0, \quad U = \begin{cases} 0, & r < R \\ U_0, & r \geq R \end{cases}$$

$$\psi(\mathbf{r}) = N_{n\kappa} \begin{pmatrix} j_l(pr) \\ -i \text{sgn}(\kappa) (\boldsymbol{\sigma} \cdot \mathbf{r}) j_{l'}(pr) \end{pmatrix} \chi$$

$$N_{n\kappa} = \left(\int_0^R dr r^2 (j_l^2(pr) + j_{l'}^2(pr)) \right)^{-\frac{1}{2}}$$

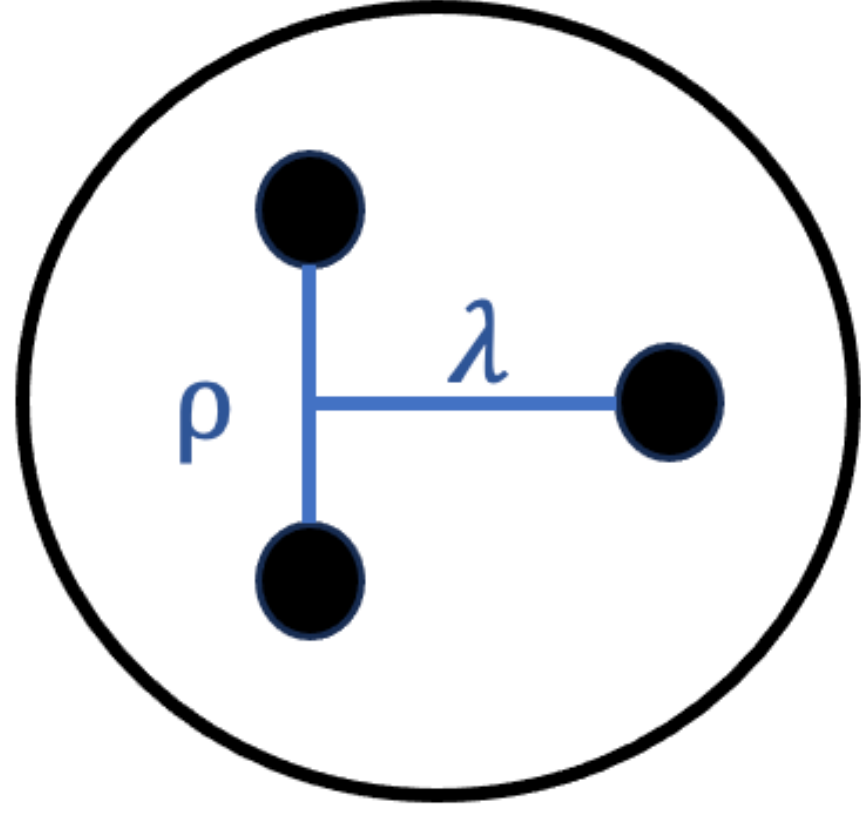
$$\psi = \begin{pmatrix} iu(r)\chi \\ v(r)\boldsymbol{\sigma} \cdot \hat{\mathbf{r}}\chi \end{pmatrix}$$

Baryon wave function:

$$|\mathcal{B}^\uparrow\rangle = \frac{1}{\sqrt{6}} \epsilon^{\alpha\beta\gamma} \int \chi_s^\uparrow \phi_{\alpha\beta\gamma}^{abc}(x_1, x_2, x_3) \Psi_{\mathcal{B}(q_1 q_2 q_3)}(x_1, x_2, x_3) [d^3x] |0\rangle$$

$$\Psi_{\mathcal{B}(q_1 q_2 q_3)}(x_1, x_2, x_3) = \psi_{q_1}(x_1) \psi_{q_2}(x_2) \psi_{q_3}(x_3)$$

NON-RELATIVISTIC QUARK MODEL



NR quark model

$$H = \sum_{i=1}^3 \frac{\mathbf{p}_i^2}{2m_i} + \frac{1}{2} K \sum_{i<j} (\mathbf{r}_i - \mathbf{r}_j)^2$$

$$\begin{aligned} \mathbf{R}_c &= \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3}{m_1 + m_2 + m_3}, \\ \boldsymbol{\rho} &= \mathbf{r}_1 - \mathbf{r}_2, \\ \boldsymbol{\lambda} &= \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \mathbf{r}_3. \end{aligned}$$

$$H = \frac{\mathbf{p}^2}{2M} + \frac{\mathbf{p}_\rho^2}{2m_\rho} + \frac{\mathbf{p}_\lambda^2}{2m_\lambda} + \frac{1}{2} m_\rho \omega_\rho^2 \boldsymbol{\rho}^2 + \frac{1}{2} m_\lambda \omega_\lambda^2 \boldsymbol{\lambda}^2$$

$$\begin{aligned} \mathbf{p} &= M \dot{\mathbf{R}}_c = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3, \\ \mathbf{p}_\rho &= m_\rho \dot{\boldsymbol{\rho}} = \frac{m_2}{m_1 + m_2} \mathbf{p}_1 - \frac{m_1}{m_1 + m_2} \mathbf{p}_2, \\ \mathbf{p}_\lambda &= m_\lambda \dot{\boldsymbol{\lambda}} = \frac{m_3(\mathbf{p}_1 + \mathbf{p}_2) - (m_1 + m_2)\mathbf{p}_3}{(m_1 + m_2 + m_3)}. \end{aligned}$$

$$\begin{aligned} |\mathcal{B}(\mathbf{P}_c)_{J,M}\rangle &= \sum_{S_z, M_L; c_i} \langle L, M_L; S, S_z | J, M \rangle \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{P}_c) \Psi_{N,L,M_L}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \\ &\quad \times \chi_{s_1, s_2, s_3}^{S, S_z} \frac{\epsilon_{c_1 c_2 c_3}}{\sqrt{6}} \phi_{i_1, i_2, i_3} b_{c_1, i_1, s_1, \mathbf{p}_1}^\dagger b_{c_2, i_2, s_2, \mathbf{p}_2}^\dagger b_{c_3, i_3, s_3, \mathbf{p}_3}^\dagger |0\rangle, \end{aligned}$$

$$\langle \mathcal{B}(\mathbf{P}'_c)_{J,M} | \mathcal{B}(\mathbf{P}_c)_{J,M} \rangle = \delta^3(\mathbf{P}'_c - \mathbf{P}_c)$$

$$\Psi_{LM_L n_\rho l_\rho n_\lambda l_\lambda}(\mathbf{P}, \mathbf{p}_\rho, \mathbf{p}_\lambda) = \delta^3(\mathbf{P} - \mathbf{P}_c) \sum_m \langle LM_L | l_\rho m, l_\lambda M_L - m \rangle \psi_{n_\rho l_\rho m}(\mathbf{p}_\rho) \psi_{n_\lambda l_\lambda (M_L - m)}(\mathbf{p}_\lambda)$$

$$\psi_{nLm}(\mathbf{p}) = (i)^l (-1)^n \left[\frac{2n!}{(n + L + \frac{1}{2})!} \right]^{\frac{1}{2}} \frac{1}{\alpha^{L+\frac{3}{2}}} e^{-\frac{\mathbf{p}^2}{2\alpha^2}} L_n^{L+\frac{1}{2}} \left(\frac{\mathbf{p}^2}{\alpha^2} \right) \mathcal{Y}_{Lm}(\mathbf{p})$$

MODEL ESTIMATIONS

$$\langle \mathcal{B}_c(p_2) | \bar{c} \gamma_\mu (1 - \gamma_5) u | \mathcal{B}_{cc}(p_1) \rangle = \bar{u}_2 \left[f_1(q^2) \gamma_\mu - f_2(q^2) i \sigma_{\mu\nu} \frac{q^\nu}{M} + f_3(q^2) \frac{q_\mu}{M} - \left(g_1(q^2) \gamma_\mu - g_2(q^2) i \sigma_{\mu\nu} \frac{q^\nu}{M} + g_3(q^2) \frac{q_\mu}{M} \right) \gamma_5 \right] u_1$$

$$a_{\mathcal{B}'\mathcal{B}} \equiv \langle \mathcal{B}' | \mathcal{H}_{\text{eff}}^{\text{PC}} | \mathcal{B} \rangle = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud}^* C_- \langle \mathcal{B}' | O_- | \mathcal{B} \rangle$$

$\mathcal{B}_{cc} \rightarrow \mathcal{B}_c$	f_1		g_1		g^A		$\langle \mathcal{B}_f O_- \mathcal{B}_i \rangle$	
	MBM	NRQM	MBM	NRQM	MBM	NRQM	MBM	NRQM
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$	$\frac{\sqrt{6}}{2} X_1$	$\frac{\sqrt{6}}{2} X_2$	$\frac{\sqrt{6}}{6} X'_1$	$\frac{\sqrt{6}}{6} X_2$	$-\frac{1}{3} Y_1$	$-\frac{1}{3} Y_2$	$4\sqrt{6} Z_1$	$4\sqrt{6} Z_2$
$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$	$\frac{\sqrt{2}}{2} X_1$	$\frac{\sqrt{2}}{2} X_2$	$\frac{5\sqrt{2}}{6} X'_1$	$\frac{5\sqrt{2}}{6} X_2$	$-\frac{1}{3} Y_1$	$-\frac{1}{3} Y_2$	$-\frac{4\sqrt{2}}{3} Z'_1$	0

Bag
model

$$X_1 = 4\pi \int_0^R r^2 dr (u_s u_c + v_s v_c)$$

0.95

$$X'_1 = 4\pi \int_0^R r^2 dr (u_s u_c - \frac{1}{3} v_s v_c)$$

0.86

$$Y_1 = 4\pi \int_0^R r^2 dr (u_u^2 - \frac{1}{3} v_u^2)$$

0.65

$$Z_1 = 4\pi \int_0^R r^2 dr (u_s u_u + v_s v_u) (u_c u_d + v_c v_d)$$

2.19×10^{-3}

$$Z'_1 = 4\pi \int_0^R r^2 dr (u_s v_u - v_s u_u) (u_c v_d - v_c u_d)$$

4.47×10^{-5}

NRQM

$$X_2 = \left(\frac{16(m_s + m_u)^2 \alpha_{\lambda 1} \alpha_{\lambda 2} \alpha_{\rho 1} \alpha_{\rho 2}}{D_1 + D_2} \right)^{3/2}$$

0.58

$$Y_2 = 8 \left(\frac{\alpha_{\lambda 1} \alpha_{\lambda 3} \alpha_{\rho 1} \alpha_{\rho 3}}{(\alpha_{\lambda 1}^2 + \alpha_{\lambda 3}^2)(\alpha_{\rho 1}^2 + \alpha_{\rho 3}^2)} \right)^{3/2}$$

1.00

$$Z_2 = 128\sqrt{2}\pi^{3/2} \left(\frac{\alpha_{\lambda 2} \alpha_{\lambda 3} \alpha_{\rho 2} \alpha_{\rho 3}}{4\alpha_{\lambda 2}^2 + \alpha_{\lambda 3}^2 + 4\alpha_{\rho 3}^2} \right)^{3/2}$$

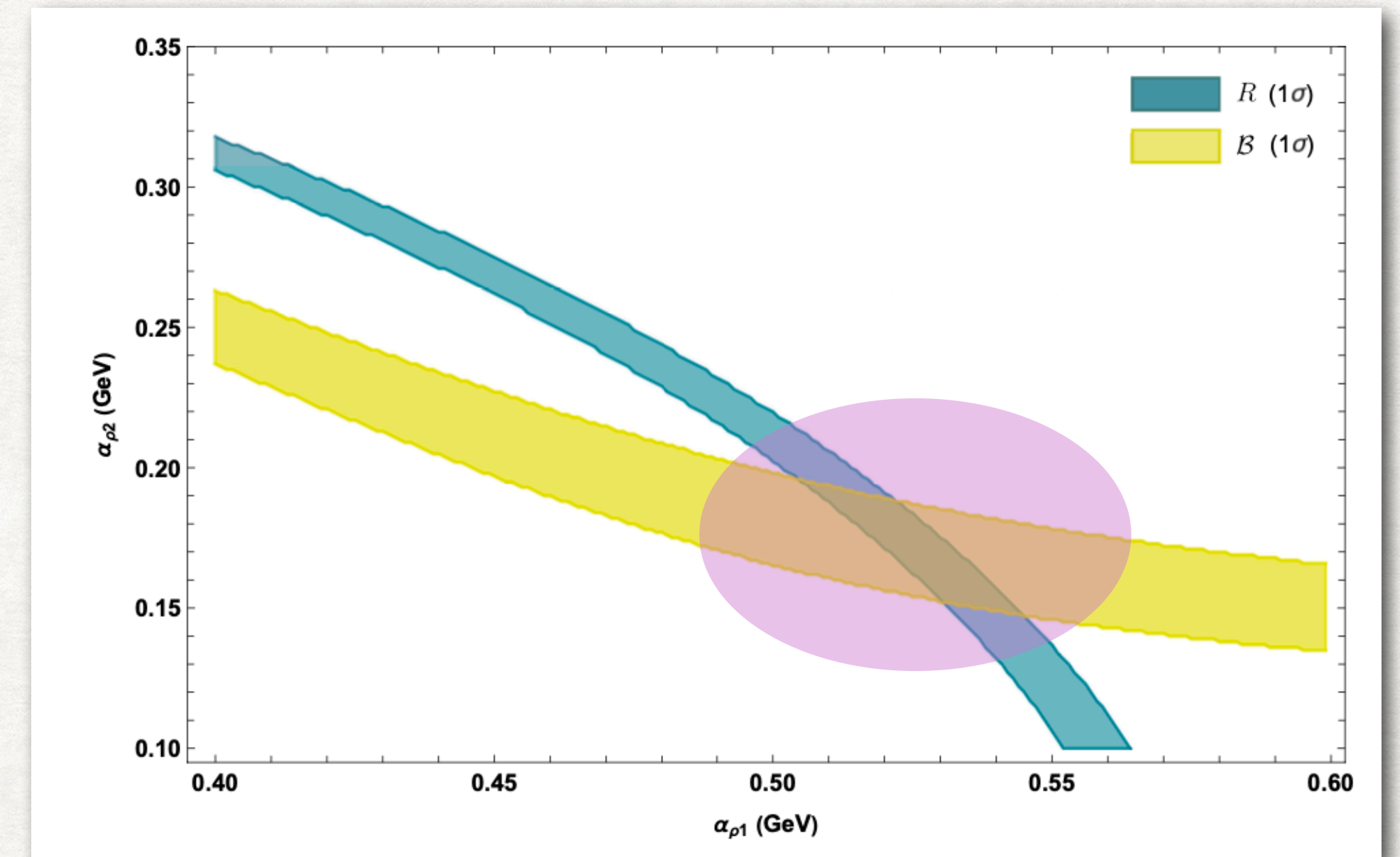
$Z_2 = 0.79$

MODEL INPUTS

$$\langle \mathcal{B}_c(p_2) | \bar{c} \gamma_\mu (1 - \gamma_5) u | \mathcal{B}_{cc}(p_1) \rangle = \bar{u}_2 \left[f_1(q^2) \gamma_\mu - f_2(q^2) i \sigma_{\mu\nu} \frac{q^\nu}{M} + f_3(q^2) \frac{q_\mu}{M} - \left(g_1(q^2) \gamma_\mu - g_2(q^2) i \sigma_{\mu\nu} \frac{q^\nu}{M} + g_3(q^2) \frac{q_\mu}{M} \right) \gamma_5 \right] u_1$$

$$a_{\mathcal{B}'\mathcal{B}} \equiv \langle \mathcal{B}' | \mathcal{H}_{\text{eff}}^{\text{PC}} | \mathcal{B} \rangle = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud}^* c_- \langle \mathcal{B}' | O_- | \mathcal{B} \rangle$$

	$(\alpha_{\rho 1}, \alpha_{\rho 2})$	$f_1(m_P^2)$	$g_1(m_P^2)$	$g_{\mathcal{B}'\mathcal{B}}^{A(\pi)}$	$\langle \mathcal{B}_f O_- \mathcal{B}_i \rangle$
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$					
MBM		0.577	0.222	-0.217	0.0214
NRQM					
Case 1	(0.50, 0.21)	0.709	0.236	-0.333	0.0310
Case 2	(0.51, 0.19)	0.574	0.191	-0.333	0.0247
Case 3	(0.53, 0.17)	0.425	0.141	-0.333	0.0191
$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$					
MBM		0.386	0.703	-0.217	8.4×10^{-5}
NRQM					
Case 1	(0.50, 0.21)	0.397	0.662	-0.333	0
Case 2	(0.51, 0.19)	0.323	0.538	-0.333	0
Case 3	(0.53, 0.17)	0.240	0.400	-0.333	0



$$[\Xi]_{cc}^{++} : \alpha_{\lambda 1} = \left[\frac{16m_u}{3(2m_c + m_u)} \right]^{\frac{1}{4}} \alpha_{\rho 1}$$

$$[\Xi]_c^+ : \alpha_{\lambda 2} = \left[\frac{4m_c(m_s + m_u)^2}{3m_s m_u(m_s + m_u + m_c)} \right]^{\frac{1}{4}} \alpha_{\rho 2}$$

$$[\Xi]_c'^+ : \alpha_{\lambda 3} = \left[\frac{16m_d}{3(2m_c + m_d)} \right]^{\frac{1}{4}} \alpha_{\rho 3}$$

RESULTS

	A^{fac}	A^{nf}	A^{tot}	B^{fac}	B^{nf}	B^{tot}	$10^2 \mathcal{B}$	α	R
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$									
MBM	7.40	-10.79	-3.38	-15.06	18.91	3.85	0.69	-0.41	
NRQM Case 1	9.1	-15.6	-6.5	-16.0	27.4	11.4	3.01	-0.78	
Case 2	7.4	-12.4	-5.0	-13.0	21.8	8.8	1.83	-0.78	
Case 3	7.4	-10.8	-3.4	-15.1	18.9	3.8	0.69	-0.41	
$\Xi_{cc}^{++} \rightarrow \Xi_c'^+ \pi^+$									
MBM	4.49	-0.04	4.45	-48.50	0.06	-48.44	4.65	-0.84	6.74
NRQM Case 1	4.6	0	4.6	-45.6	0	-45.6	4.32	-0.89	1.44
Case 2	3.7	0	3.7	-37.1	0	-31.0	2.86	-0.89	1.56
Case 3	2.8	0	2.8	-27.6	0	-27.6	2.16	-0.89	1.32
LHCb							1.41 ± 0.20		

$$\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+)_{\text{expt}} \approx (1.33 \pm 0.74)\%$$

- NF amplitudes are suppressed for $\Xi_c'^+ \pi^+$ mode, due to Korner-Pati-Woo theorem.
- Both S- and P-wave NF amplitudes in $\Xi_c^+ \pi^+$ dominate and have destructive interference with factorizable ones
- MIT bag model estimation gives small $\Xi_c^+ \pi^+$ and large ratio R.
- NR quark model estimation can reach experimental requirement, taking Case 2 as an example.
- The sizes of non-factorizable amplitudes: bag model < NRQM.

[15] J.G. Körner, Octet behaviour of single-particle matrix elements $\langle B' | H_W | B \rangle$ and $\langle M' | H_W | M \rangle$ using a weak current-current quark Hamiltonian, *Nucl. Phys. B* **25** (1971) 282 [INSPIRE].
[16] J.C. Pati and C.H. Woo, $\Delta I = \frac{1}{2}$ rule with fermion quarks, *Phys. Rev. D* **3** (1971) 2920 [INSPIRE].

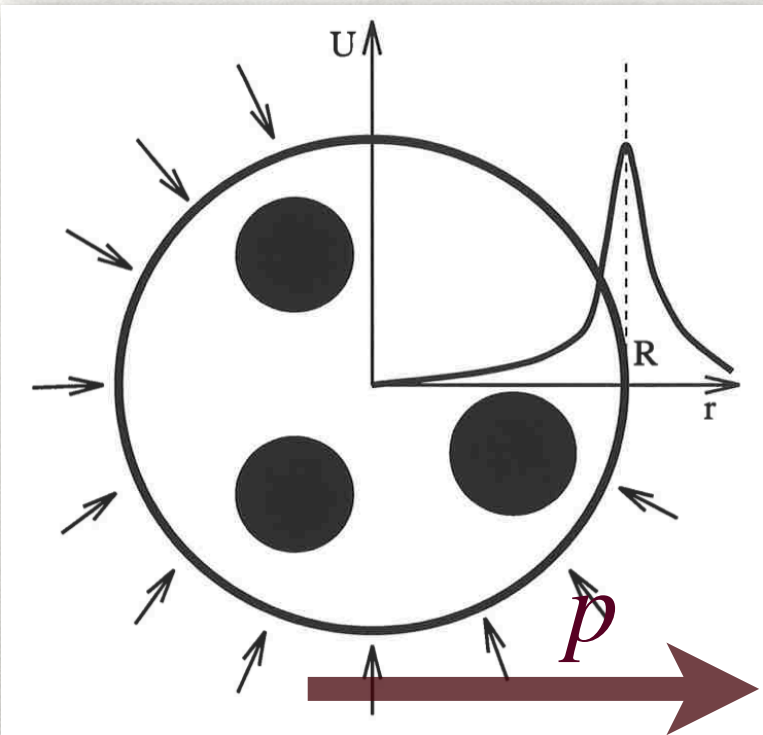
COMPARISON

	A^{fac}	A^{nf}	A^{tot}	B^{fac}	B^{nf}	B^{tot}	\mathcal{B}	α	R
$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$									
MBM [11]	7.4	-10.8	-3.4	-15.1	18.9	3.8	0.69	-0.41	
NRQM									
Case 1	9.1	-15.6	-6.5	-16.0	27.4	11.4	3.01	-0.78	
Case 2	7.4	-12.4	-5.0	-13.0	21.8	8.8	1.83	-0.78	
Case 3	5.5	-9.6	-4.1	-9.6	16.8	7.2	1.20	-0.78	
Gutsche <i>et al.</i> [5]	-8.1	11.5	3.4	13.0	-18.5	-5.6	0.71	-0.57	
Sharma & Dhir [8]									
NRQM	7.38	0	7.38	-16.77	-24.95	-41.72	6.64	-0.99	
HQET	9.52	0	9.52	-19.45	-24.95	-44.40	9.19	-0.99	
Shi <i>et al.</i> [6]									
LCSR+HQET	9.52	-16.67	-7.18	-19.45	-20.47	-39.92	6.22	+0.99	
Ke & Li [9]									
$\theta = 16.27^\circ$							2.14	-0.09	
$\theta = 85.54^\circ$							2.14	-0.95	
Liu & Geng [31] ⁴									
SB ($\theta = -24.7^\circ$)	4.83	-9.99	-5.16	5.16	13.6	18.8	2.24	-0.93	Ξ_c mixing
HB ($\theta = 24.7^\circ$)	7.08	-20.3	-13.2	-22.1	33.0	10.9	10.3	-0.30	

destructive

constructive

- current data on Br and R
- future measurement of α help to discriminate theoretical studies



SUMMARY

- It is the time to study charmed baryon decays.
- The decays $\Xi_{cc}^{++} \rightarrow \Xi_c^{(\prime)+} \pi^+$ have been studied theoretically in the pole model with the assistance of topological diagrams.
- Non-factorizable contributions play an essential role in charmed baryon decays.
- Two quark model have been investigated in the calculation of non-perturbative parameters contributing to non-factorizable amplitudes.
- Current data (R and one branching fraction) can be explained in proper parameter space of NR quark model or bag model with sizable Ξ_c mixing.
- More experimental and theoretical progresses are anticipated.

Thank you for your attention!

BACKUP SLIDES

$\Xi_c - \Xi'_c$ mixing

$$R = 2\Gamma(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) / 3\Gamma(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e)$$

$$\mathcal{B}_{\text{Belle}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%,$$

$$\mathcal{B}_{\text{ALICE}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.43 \pm 0.25 \pm 0.35 \pm 0.72)\%,$$

$$\mathcal{B}_{\text{LQCD}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.38 \pm 0.44)\%$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) = (3.56 \pm 0.11 \pm 0.07)\%$$

BESIII

$$R(\text{Belle}) = 0.33 \pm 0.10, \quad R(\text{ALICE}) = 0.60 \pm 0.21, \quad R(\text{LQCD}) = 0.59 \pm 0.11$$

$$R'_{av} = 0.46 \pm 0.07, \quad R_{av} = 0.59 \pm 0.10$$

$$R(SU(3)_F) = 1$$

$$|\theta_c| = 0.137(5)\pi$$

$$0.430398$$

$$|\Xi_c\rangle = \cos \theta_c |\Xi_c^{\bar{3}}\rangle + \sin \theta_c |\Xi_c^6\rangle$$

$$|\Xi'_c\rangle = \cos \theta_c |\Xi_c^6\rangle - \sin \theta_c |\Xi_c^{\bar{3}}\rangle$$

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

$$\theta = (1.220 \pm 0.130 \pm 0.010)^\circ$$

C.Q. Geng, X.-N. Jin, C.-W. Liu, PLB 833 (2023) 137736

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