Rare $D^0 \to \pi^+\pi^-\ell^+\ell^-$ decays: Contribution of S-wave dynamics in the SM and sensitivity to New Physics

based on 2308.XXXXX

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Introduction

Flavor Physics with rare processes

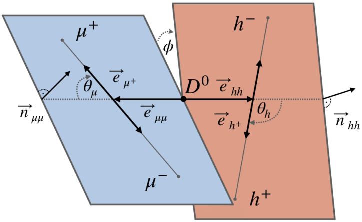
- Rare decays could provide the ground for discovering New Physics effects: SM at loop level
- Rich history: $K \to \mu\mu$, B Physics
- Many NP tests can be constructed
- Some tests theoretically very clean (e.g. LFU tests)
- Others sensitive to hadronic effects - require careful QCD implementation (e.g. in P_5')
- Rare decays with charm provide complementary tests from the up-type sector

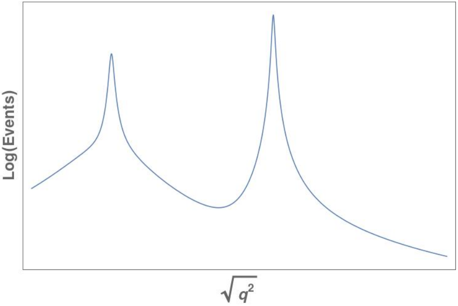
See e.g. Burdman et al., Fajfer et al.,Bharucha et al.,...

Rare processes with charm

New LHCb results on $D^0 \to \pi^+\pi^-\mu^+\mu^-$ and $D^0 \to K^+K^-\mu^+\mu^-$ [PRL 128(22):221801]

- 5D phase space: q^2 (leptons),p^2 (pions), $\theta_h, \theta_\ell, \phi$
  $\Rightarrow$ 5 1D plots of differential decay rates (LHCb data for q^2, p^2)





- Angular observables: different integrations
  $\sum_i c_i \int_{\theta_{\ell,i}}^{\theta_{\ell,i+1}} d\cos\theta_\ell \sum_j c'_j \int_{\phi_i}^{\phi_{i+1}} d\phi d^5\Gamma$ (e.g. A_{FB} is a null test!)
- CP-asymmetric ones: further tests (not the focus of the talk)

Rare processes with charm

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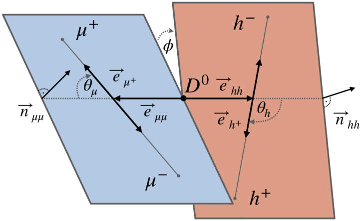


Table 2: Angular observables $\langle S_i \rangle$ for (top) $D^0 \to \pi^+\pi^-\mu^+\mu^-$ and (bottom) $D^0 \to K^+K^-\mu^+\mu^-$ decays in the dimuon-mass regions. The first uncertainty is statistical and the second systematic.

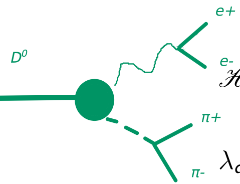
$m(\mu^+\mu^-)$ [MeV/c^2]	$\langle S_2 \rangle$ [%]	$\langle S_3 \rangle$ [%]	$\langle S_4 \rangle$ [%]	$\langle S_5 \rangle$ [%]	$\langle S_6 \rangle$ [%]	$\langle S_7 \rangle$ [%]	$\langle S_8 \rangle$ [%]	$\langle S_9 \rangle$ [%]
				$D^0 \to \pi^+\pi^-\mu^+\mu^-$				
< 525	$5 \pm 14 \pm 4$	$-6 \pm 16 \pm 2$	$21 \pm 16 \pm 2$	$-20 \pm 14 \pm 1$	$-14 \pm 14 \pm 1$	$8 \pm 14 \pm 1$	$16 \pm 17 \pm 1$	$26 \pm 16 \pm 2$
525–565	–	–	–	–	–	–	–	–
565–780	$-2.4 \pm 4.1 \pm 1.1$	$-9.1 \pm 4.8 \pm 1.5$	$3.7 \pm 4.9 \pm 1.3$	$-3.0 \pm 4.1 \pm 0.8$	$2.5 \pm 4.1 \pm 0.6$	$0.8 \pm 4.1 \pm 1.0$	$12.9 \pm 4.9 \pm 1.0$	$-0.1 \pm 4.9 \pm 0.9$
780–950	$-10.7 \pm 5.8 \pm 1.1$	$7.7 \pm 6.9 \pm 1.0$	$-4.7 \pm 6.9 \pm 1.5$	$4.7 \pm 5.8 \pm 0.7$	$9.0 \pm 5.8 \pm 0.7$	$-4.7 \pm 5.8 \pm 1.0$	$1.4 \pm 6.9 \pm 0.7$	$-4.7 \pm 6.8 \pm 0.8$
950–1020	$-2.0 \pm 3.7 \pm 1.6$	$-17.4 \pm 4.3 \pm 1.5$	$-9.9 \pm 4.3 \pm 3.5$	$2.0 \pm 3.7 \pm 0.8$	$6.5 \pm 3.7 \pm 1.4$	$-3.6 \pm 3.7 \pm 1.1$	$2.6 \pm 4.3 \pm 0.9$	$16.9 \pm 4.3 \pm 1.0$
1020–1100	$1.7 \pm 3.4 \pm 1.5$	$-15.3 \pm 4.0 \pm 1.7$	$-18.3 \pm 4.0 \pm 2.5$	$-6.9 \pm 3.4 \pm 1.2$	$1.1 \pm 3.4 \pm 0.8$	$2.7 \pm 3.4 \pm 2.0$	$0.7 \pm 4.1 \pm 0.9$	$7.8 \pm 4.0 \pm 1.7$
> 1100	–	– –	–	–	–	–	–	
Full range	$-3.4 \pm 2.1 \pm 1.0$	$-10.4 \pm 2.5 \pm 0.9$	$-4.6 \pm 2.5 \pm 1.6$	$-2.9 \pm 2.1 \pm 0.6$	$3.7 \pm 2.1 \pm 0.5$	$-0.6 \pm 2.1 \pm 0.9$	$3.8 \pm 2.5 \pm 0.5$	$5.1 \pm 2.5 \pm 0.5$

Table 3: Angular observables $\langle A_i \rangle$ for (top) $D^0 \to \pi^+\pi^-\mu^+\mu^-$ and (bottom) $D^0 \to K^+K^-\mu^+\mu^-$ decays in the dimuon-mass regions. The first uncertainty is statistical and the second systematic.

$m(\mu^+\mu^-)$ [MeV/c^2]	$\langle A_2 \rangle$ [%]	$\langle A_3 \rangle$ [%]	$\langle A_4 \rangle$ [%]	$\langle A_5 \rangle$ [%]	$\langle A_6 \rangle$ [%]	$\langle A_7 \rangle$ [%]	$\langle A_8 \rangle$ [%]	$\langle A_9 \rangle$ [%]
				$D^0 \to \pi^+\pi^-\mu^+\mu^-$				
< 525	$-10 \pm 14 \pm 2$	$2 \pm 16 \pm 1$	$-7 \pm 16 \pm 2$	$16 \pm 14 \pm 1$	$0 \pm 14 \pm 1$	$-10 \pm 14 \pm 2$	$3 \pm 17 \pm 2$	$-25 \pm 16 \pm 2$
525–565	–	–	–	–	–	–	–	–
565–780	$-1.1 \pm 4.1 \pm 1.9$	$5.7 \pm 4.8 \pm 0.7$	$0.6 \pm 4.9 \pm 0.7$	$-3.0 \pm 4.1 \pm 1.1$	$-4.8 \pm 4.1 \pm 1.0$	$-3.5 \pm 4.1 \pm 1.0$	$-1.8 \pm 4.9 \pm 1.2$	$1.6 \pm 4.9 \pm 1.1$
780–950	$-7.7 \pm 5.8 \pm 0.6$	$3.9 \pm 6.9 \pm 0.8$	$1.2 \pm 6.9 \pm 0.7$	$-3.3 \pm 5.8 \pm 1.0$	$0.4 \pm 5.8 \pm 1.0$	$-2.6 \pm 5.8 \pm 0.6$	$-5.1 \pm 6.9 \pm 1.5$	$-2.9 \pm 6.8 \pm 1.0$
950–1020	$2.3 \pm 3.7 \pm 0.7$	$-2.2 \pm 4.3 \pm 2.1$	$7.6 \pm 4.3 \pm 0.9$	$-3.6 \pm 3.7 \pm 1.2$	$4.5 \pm 3.7 \pm 1.1$	$3.5 \pm 3.7 \pm 0.9$	$2.7 \pm 4.3 \pm 1.3$	$1.4 \pm 4.3 \pm 1.2$
1020–1100	$-4.8 \pm 3.4 \pm 0.9$	$-2.6 \pm 4.0 \pm 1.2$	$-2.4 \pm 4.0 \pm 1.0$	$-2.3 \pm 3.4 \pm 1.2$	$3.2 \pm 3.4 \pm 1.1$	$-1.3 \pm 3.4 \pm 0.8$	$5.1 \pm 4.1 \pm -1.3$	$-5.9 \pm 4.0 \pm 1.8$
> 1100	–	– –	–	–	–	–	–	
Full range	$-2.6 \pm 2.1 \pm 0.7$	$0.9 \pm 2.5 \pm 0.5$	$1.3 \pm 2.5 \pm 0.5$	$-1.7 \pm 2.1 \pm 0.9$	$-0.5 \pm 2.1 \pm 0.9$	$-1.5 \pm 2.1 \pm 0.6$	$0.3 \pm 2.5 \pm 1.0$	$-2.1 \pm 2.5 \pm 0.9$

- Angular observables: different integrations
  $\sum_i c_i \int_{\theta_{\ell,i}}^{\theta_{\ell,i+1}} d\cos\theta_\ell \sum_j c'_j \int_{\phi_i}^{\phi_{i+1}} d\phi d^5\Gamma$ (e.g. A_{FB} is a null test!)
- CP-asymmetric ones: further tests (not the focus of the talk)

What is interesting about rare charm decays?

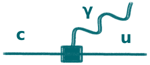


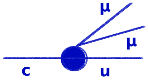
$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\Big[\sum_{i=1}^{2} C_i\left(\lambda_d Q_i^d + \lambda_s Q_i^s\right) - \lambda_b\left(C_7 Q_7 + C_9 Q_9 + C_{10} Q_{10}\right)\Big]$$

$$\lambda_q = V_{cq}^* V_{uq}\,, \qquad q = d, s, b\,.$$

$$Q_1^d = (\bar{d}c)_{V-A}(\bar{u}d)_{V-A}\,,$$
$$Q_2^d = (\bar{u}c)_{V-A}(\bar{d}d)_{V-A}\,,$$
$$Q_1^s = (\bar{s}c)_{V-A}(\bar{u}s)_{V-A}\,,$$
$$Q_2^s = (\bar{u}c)_{V-A}(\bar{s}s)_{V-A}\,,$$

$$Q_7 = -\frac{g_{em}}{8\pi^2} m_c \bar{u}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}c\,,$$
$$Q_9 = \frac{\alpha_{em}}{2\pi}(\bar{u}\gamma_\mu(1-\gamma_5)c)(\bar{\ell}\gamma^\mu\ell)\,,$$
$$Q_{10} = \frac{\alpha_{em}}{2\pi}(\bar{u}\gamma_\mu(1-\gamma_5)c)(\bar{\ell}\gamma^\mu\gamma_5\ell)\,,$$



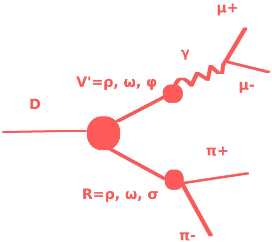


- Strong GIM suppression $\Rightarrow C_{9,10,7}$ very small! ($\neq B$ Physics:$|C_9|, |C_{10}| \approx 4.2$)
- Result:
  1. (+) Some observables vanish in the SM ("null tests") [de Boer, Hiller Phys.Rev.D 98 (2018) 3]
  2. (-) SM contributions are only long-distance/non-perturbative QCD $\Rightarrow$ difficult territory to navigate!
  3. Typical picture: interplay between SM and potential NP contributions
  4. To probe NP Wilson coefficients of e.g. SMEFT, we need good control of SM contributions

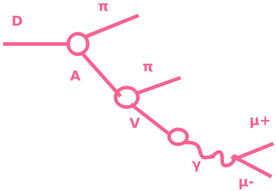
Framework

Rare D decays: model

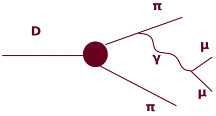
Quasi-two-body



Cascade



Bremmstrahlung



Rare D decays: model

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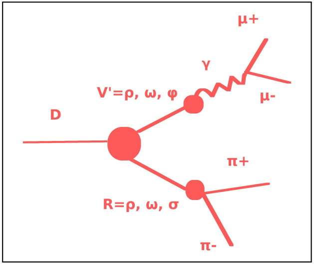
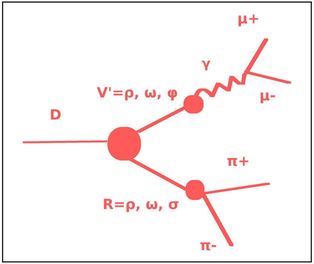


$$\langle \pi^+ \pi^- \ell^+ \ell^- | S | D^0 \rangle = \langle \pi^+ \pi^- \ell^+ \ell^- | \int d^4x\, d^4w\, d^4y\, d^4z \sum_{\mathscr{R},\mathscr{V}} T\{ \mathscr{H}_{em}^{\text{lept}}(z)\, \mathscr{H}_{\mathscr{V}\gamma}(y)\, \mathscr{H}_{\mathscr{R}\pi\pi}(w)\, \mathscr{H}_{eff}(x) \} | D^0 \rangle$$

Rare D decays: model

Quasi-two-body



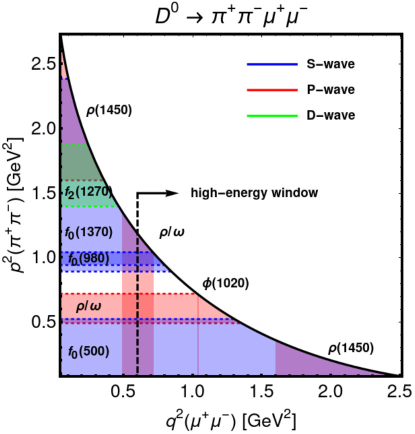
$$\langle \pi^+\pi^-\ell^+\ell^- | S | D^0 \rangle = \langle \pi^+\pi^-\ell^+\ell^- | \int d^4x\, d^4w\, d^4y\, d^4z \sum_{\mathcal{R},\mathcal{V}} T\{\mathcal{H}_{em}^{\mathrm{lept}}(z)\, \mathcal{H}_{\mathcal{V}\gamma}(y)\, \mathcal{H}_{\mathcal{R}\pi\pi}(w)\, \mathcal{H}_{eff}(x)\} | D^0 \rangle$$

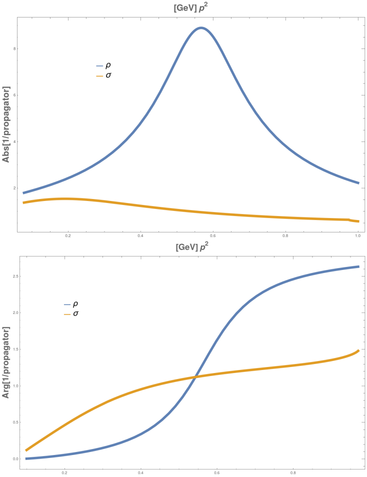
New: we include the scalar resonance $f_0(500) = \sigma$ in $D^0 \to \sigma \to \pi^+\pi^-$

(observed in $D \to \pi\pi e \nu_e$ [BESIII], evidence in $D \to 4\pi$ [d'Argent+, CLEO data])

Instead of Breit-Wigner, Bugg's parameterization [Bugg, Phys.Lett.B 572 (2003) 1-7]: includes rescattering effects

Resonances


$D^0 \to \pi^+\pi^-\mu^+\mu^-$
S–wave
P–wave
D–wave
high–energy window
ρ(1450)
f2(1270)
f0(1370)
f0(980)
ρ/ω
ϕ(1020)
ρ/ω
f0(500)
ρ(1450)
$p^2(\pi^+\pi^-)$ [GeV^2]
$q^2(\mu^+\mu^-)$ [GeV^2]


[GeV] p^2
ρ
σ
Abs[1/propagator]
[GeV] p^2
ρ
σ
Arg[1/propagator]

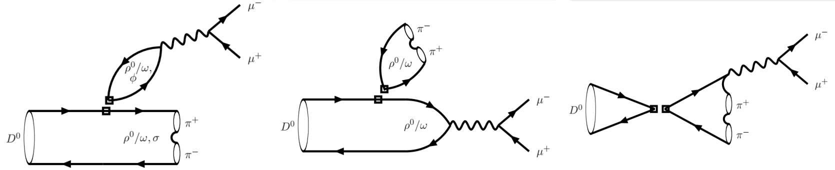
Factorisation-inspired

Starting point: Cappiello, Cata, d'Ambrosio, JHEP 04 (2013) 135

$$Q_2^d = (\bar{u}c)_{V-A}(\bar{d}d)_{V-A}$$
$$Q_2^s = (\bar{u}c)_{V-A}(\bar{s}s)_{V-A}$$

$\rho, \omega, \sigma : \{u, d\}, \quad \phi : s$ Zweig rule

Tree -type 1 ('W')	Tree -type 2 ('J')	Annihilation
$D \to R \to \pi\pi$ $0 \to V \to \ell\ell$	$D \to V \to \ell\ell$ $0 \to R \to \pi\pi$	$D \to 0$ $0 \to R(\to \pi\pi)V(\to \ell\ell)$



- J-type important: same CKM powers as W-type, unflavored final mesons ($\neq$ B-Physics)
- Correct factorization with the introduction of strong phases and 'fudge factors'
- Annihilation: only $\rho\rho$, $\sigma\rho$ - $\sim m_d$, partially reabsorbed in free parameters

Angular observables I

Defined e.g. as

$$I_2 = \int_{-\pi}^{\pi} d\phi \left[\int_{-1}^{-0.5} d\cos\theta_\ell + \int_{0.5}^{1} d\cos\theta_\ell - \int_{-0.5}^{0.5} d\cos\theta_\ell \right] \frac{d^5\Gamma}{dp^2 dq^2 d\Omega}$$

results in

$$I_i = f(\underbrace{C_9^{eff:P}(q^2), C_9^{eff:S}(q^2)}_{\text{SM, P- and S- wave}}; \underbrace{C_9^{NP}, C_{10}^{NP}, C_9^{\prime NP}, C_{10}^{\prime NP}}_{\text{NP only}}) \times (\text{Long-distance part})$$

Null tests

I_5, I_6 (Forward-backward asymmetry) , $I_7 \stackrel{SM}{=} 0$

Further integration over θ_h

$$\langle I_i \rangle_- \equiv \left[\int_0^{+1} d\cos\theta_{P_1} - \int_{-1}^{0} d\cos\theta_{P_1} \right] I_i \,, \quad \langle I_i \rangle_+ \equiv \int_{-1}^{+1} d\cos\theta_{P_1} I_i$$

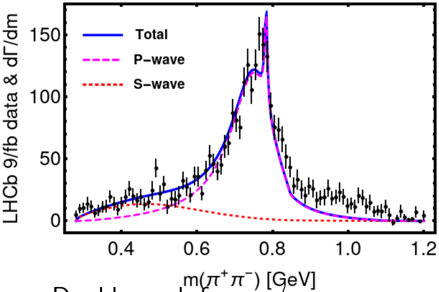
results in observables that either

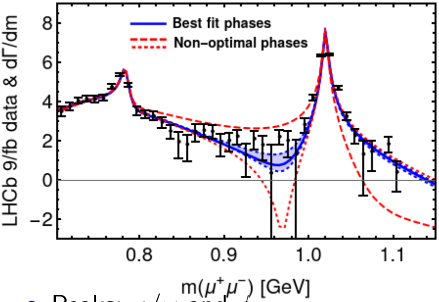
- depend on the P-wave only: $\langle I_3\rangle_+, \langle I_6\rangle_+, \langle I_9\rangle_+, \langle I_4\rangle_-, \langle I_5\rangle_-, \langle I_7\rangle_-, \langle I_8\rangle_-$
- depend on the P-wave and the S-wave but not their interference: $\langle I_1\rangle_+$, $\langle I_2\rangle_+$
- depend on the interference of S and P wave (rest of the observables)

CP-averaged ones $\langle S_i\rangle$; no CP violation $\rightarrow \langle S_i\rangle = \langle I_i\rangle_\pm$ (q^2-binned)

Results (preliminary)

Differential mass distributions: p^2, q^2





- Double peak from ρ/ω
- We fit the relative phase of $\rho - \omega$: dynamics specific to this decay (not a semileptonic one)

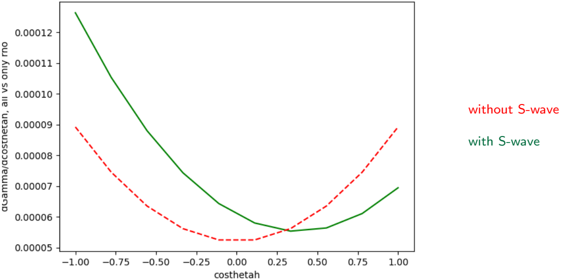
Inclusion of σ is paramount!

  $\chi^2_{min;\mathrm{w/o}\ \sigma} - \chi^2_{min} = (8.6)^2$

- Peaks: ρ/ω and ϕ
- We fit for the phase difference of ρ/ω and ϕ; significantly affects high q^2
- σ contributes to the diff. decay rate by $10 - 35\%$ depending on q^2 (not visible)

- q^2-binned observables I_2, I_3 reproduced well ; S-wave contributes more than 30% to I_2
- I_4 not reproducible with current framework

Tests for S-wave: $d\Gamma/d\cos\theta_h$, angular observables



(not yet experimentally provided)

$$\frac{d\Gamma}{d\cos\theta_h} \propto \quad |\mathscr{F}_S|^2 + |\mathscr{F}_{0,P}|^2\cos^2\theta_h + \left(|\mathscr{F}_\parallel|^2 + |\mathscr{F}_\perp|^2\right)\sin^2\theta_h$$
$$+\mathrm{Re}\{\mathscr{F}_S\mathscr{F}^*_{0,P}\}\cos\theta_h$$

Integration by p^2 regions would further highlight the presence of σ / help determine strong dynamics

Observables $\langle I_2\rangle_-$, $\langle I_4\rangle_+$, $\langle I_8\rangle_+$ calculated to be at a measurable level

Angular observables: NP

Null tests

I_5, I_6 (Forward-backward asymmetry) , $I_7 \overset{SM}{=} 0$

Examine the case of $C_{10} \neq 0,\ C_9, C_9', C_{10}' = 0$

- NP observables sensitive to P-wave only: $\langle I_5\rangle_-, \langle I_6\rangle_+, \langle I_7\rangle_-$
  1. $\langle I_5\rangle_-, \langle I_6\rangle_+$ up to a few % for current upper bounds on C_{10}
  2. $\langle I_7\rangle_- \approx 0$ very suppressed with the current long-distance parameterization (same for $\langle I_8\rangle_-, \langle I_9\rangle_+$)
- NP observables with S-wave:
  1. $\langle I_5\rangle_+$ (S-P interf.) $\approx \mathcal{O}(\langle I_5\rangle_-)$ (P-wave)
  2. $\langle I_7\rangle_+$ (S-P interf.) at the same level as $\langle I_5\rangle_+$, $>> \langle I_7\rangle_-$

Conclusions

Summary

- Rare D decays are dominated by long-distance QCD dynamics; focus on $D^0 \to \pi^+\pi^-\mu^+\mu^-$
- Factorisation-inspired + phases + room for magnitude adjustments
- New: inclusion of S-wave dynamics through the $\sigma = f_0(500)$ resonance, encoding rescattering in the relevant energy range

Results:

- Predictions for $d\Gamma/dp^2, dq^2$ (dihadron/dilepton mass) significantly improved; good agreement with data for most observables; determination of relative strong phases
- S-wave presence very prominent in decay rate over pion angle; accessible observables can probe it further
- S-wave inclusion gives access to complementary NP tests
- Some angular observables not correctly reproduced, regardless of S-wave presence; refinement (e.g. cascade decays?) needed before drawing conclusions regarding NP

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Thank you!

BACKUP

Angular observables - definition

$$I_2 = \int_{-\pi}^{\pi} d\phi \left[\int_{-1}^{-0.5} d\cos\theta_\mu + \int_{0.5}^{1} d\cos\theta_\mu - \int_{-0.5}^{0.5} d\cos\theta_\mu \right] \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_3 = \frac{3\pi}{8} \left[\int_{-\pi}^{-\frac{3\pi}{4}} d\phi + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\phi + \int_{\frac{3\pi}{4}}^{\pi} d\phi - \int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} d\phi - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\phi \right] \int_{-1}^{1} d\cos\theta_\mu \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_4 = \frac{3\pi}{8} \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi - \int_{-\pi}^{-\frac{\pi}{2}} d\phi - \int_{\frac{\pi}{2}}^{\pi} d\phi \right] \left[\int_{0}^{1} d\cos\theta_\mu - \int_{-1}^{0} d\cos\theta_\mu \right] \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_5 = \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi - \int_{-\pi}^{-\frac{\pi}{2}} d\phi - \int_{\frac{\pi}{2}}^{\pi} d\phi \right] \int_{-1}^{1} d\cos\theta_\mu \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_6 = \int_{-\pi}^{\pi} d\phi \left[\int_{0}^{1} d\cos\theta_\mu - \int_{-1}^{0} d\cos\theta_\mu \right] \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_7 = \left[\int_{0}^{\pi} d\phi - \int_{-\pi}^{0} d\phi \right] \int_{-1}^{1} d\cos\theta_\mu \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_8 = \frac{3\pi}{8} \left[\int_{0}^{\pi} d\phi - \int_{-\pi}^{0} d\phi \right] \left[\int_{0}^{1} d\cos\theta_\mu - \int_{-1}^{0} d\cos\theta_\mu \right] \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} ,$$

$$I_9 = \frac{3\pi}{8} \left[\int_{-\pi}^{-\frac{\pi}{2}} d\phi + \int_{0}^{\frac{\pi}{2}} d\phi - \int_{-\frac{\pi}{2}}^{0} d\phi - \int_{\frac{\pi}{2}}^{\pi} d\phi \right] \int_{-1}^{1} d\cos\theta_\mu \frac{d^5\Gamma}{dq^2\, dp^2\, d\vec{\Omega}} .$$

Numerical results - comparison to experiment

$\sqrt{q^2}$ region	$\Gamma[10^{-5}]$	$\frac{\Gamma_\sigma}{\Gamma}(\%)$	$(\int dp^2 I_2)_{+,\sigma}/(\int dp^2 I_2)_{+,\text{total}}(\%)$
$r^{(\text{low})}$	0.08	17	26
$r^{(\eta)}$	0.03	15	28
$r^{(\rho:\text{inf})}$	1.09	32	61
$r^{(\rho:\text{sup})}$	0.59	37	74
$r^{(\phi:\text{inf})}$	1.00	27	79
$r^{(\phi:\text{sup})}$	0.85	22	75
$r^{(\text{high})}$	0.01	13	62

$\sqrt{q^2}$ region	$\langle S_2\rangle(\%)$	$\langle S_3\rangle(\%)$	$\langle S_4\rangle(\%)$
$r^{(\text{low})}$	-15	-2	15
$r^{(\eta)}$	-17	-4	23
$r^{(\rho:\text{inf})}$	-17	-6	21
$r^{(\rho:\text{sup})}$	-17	-7	20
$r^{(\phi:\text{inf})}$	-12	-12	25
$r^{(\phi:\text{sup})}$	-10	-13	27
$r^{(\text{high})}$	-11	-23	44

$\langle S_2\rangle$ [%]	$\langle S_3\rangle$ [%]	$\langle S_4\rangle$ [%]
$5 \pm 14 \pm 4$	$-6 \pm 16 \pm 2$	$21 \pm 16 \pm 2$
–	–	–
$-2.4 \pm 4.1 \pm 1.1$	$-9.1 \pm 4.8 \pm 1.5$	$3.7 \pm 4.9 \pm 1.3$
$-10.7 \pm 5.8 \pm 1.1$	$7.7 \pm 6.9 \pm 1.0$	$-4.7 \pm 6.9 \pm 1.5$
$-2.0 \pm 3.7 \pm 1.6$	$-17.4 \pm 4.3 \pm 1.5$	$-9.9 \pm 4.3 \pm 3.5$
$1.7 \pm 3.4 \pm 1.5$	$-15.3 \pm 4.0 \pm 1.7$	$-18.3 \pm 4.0 \pm 2.5$
– –	–	–

New Physics observables

$\sqrt{q^2}$ region	Γ (SM+NP) (10^{-5})	$\langle I_5 \rangle_{(+,S)}$(%)	$\langle I_5 \rangle_{(-,P)}$(%)	$\langle I_6 \rangle_{(+,P)}$(%)	$\langle I_7 \rangle_{(+,S)}$(%)
0.212-0.525	$0.08 + 0.09\vert\tilde{C}_{10}\vert^2$	2.1	-2.4	-1.8	-2.1
0.525-0.565	$0.03 + 0.02\vert\tilde{C}_{10}\vert^2$	2.2	-2.6	-2.6	-2.0
0.565-0.78	$1.09 + 0.11\vert\tilde{C}_{10}\vert^2$	0.7	0.3	0.4	-0.9
0.78- 0.95	$0.59 + 0.10\vert\tilde{C}_{10}\vert^2$	-0.2	2.0	2.7	-0
0.95-1.02	$1.00 + 0.04\vert\tilde{C}_{10}\vert^2$	0.3	-0.3	-0.5	0.4
1.02-1.1	$0.85 + 0.03\vert\tilde{C}_{10}\vert^2$	0.02	0.5	0.9	-0.1
1.1-1.59	$0.01 + 0.03\vert\tilde{C}_{10}\vert^2$	-1.8	4.6	8.3	-0.2

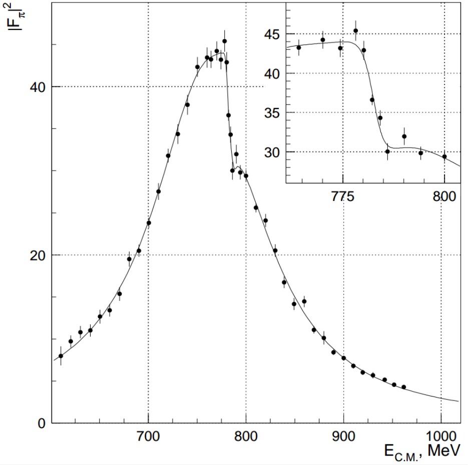
Angular observables given at $\tilde{C}_{10} = 0.43$

SM S-wave observables

For a certain choice of relative phases:

$\sqrt{q^2}$ region (GeV)	$\langle I_2 \rangle_{(+,S)}(\%)$	$\langle I_4 \rangle_{(+,S)}(\%)$	$\langle I_8 \rangle_{(+,S)}(\%)$
0.212-0.525	10	-3	2
0.525-0.565	11	-4	3
0.565-0.78	-1	2	4
0.78- 0.95	-10	6	2
0.95-1.02	14	-9	1
1.02-1.1	-5	3	4
1.1-1.59	-20	15	2

CMD-2



Compare to CMD-2: