Understanding Charm with LCSR using $D^* \to D\gamma$ Decays

(Based on ongoing work)

In collaboration with Prof. Namit Mahajan

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20/07/2023

Outline

- Motivation
- Introduction to $D_q^* D_q \gamma$ coupling
- Light Cone Sum Rules in a Nutshell
- $D_q^* D_q \gamma$ coupling in LCSR
- Results (preliminary)
- Summary and discussion

Motivation

- Distribution amplitudes (DAs) are very crucial universal non-perturbative input for theoretical computations.
- DAs for heavy meson case are modelled using the heavy quark expansion. No precise form is known so far.
  [Grozin and Neubert, PRD 55 (1997) 272-290]
- Exclusive decays indicate that the first inverse moment of these DAs is the most important parameter.

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- For B-meson case, λ_B has large uncertainty: Ranges from $\lambda_B \approx 0.2$ GeV (favoured by Non-leptonic decays) to (0.45 ± 0.15) GeV (obtained using QCD sum rule calculations).

[Beneke et. Al, Nucl. Phys. B 675 (2003) 333,
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- Study of radiative mode ($B^- \to \ell^- \nu_\ell \gamma$) is the simplest process suggested for the study of λ_B : provides only the constraints ($\lambda_B > 0.3$ MeV).

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Our Objective

To look for some other mode which can help us get the estimates for the inverse moment: $D_q^* \to D_q \gamma$

$D_q^* \to D_q \gamma$ Decays: An Introduction

- The amplitude for $D_q^* \to D_q \gamma$ $(q = u, d, s)$ is:

Coupling

$$\mathscr{M}(D_q^* \to D_q(p)\gamma(k)) = e g_{D_q} \epsilon_{\mu\nu\rho\sigma} k^\rho \epsilon_\gamma^\sigma v^\nu \epsilon_{D_q^*}^\mu$$

$\frac{e g_{D_q}}{2}$ is the transition magnetic moment.

- The decay width: $$\Gamma(D_q^*(p') \to D_q(p)\gamma(k)) = \frac{\alpha_{em}}{3} |g_{D_q}|^2 |\vec{k}|^3$$

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Experimental Data

Channel	Branching Ratio	Decay widths	g_{D_q} (Calculated)
$D^{*+} \to D^+\gamma$ $(q = d)$	(1.6 ± 0.4) %	(83.4 ± 1.8) KeV	0.47 ± 0.06
$D^{*0} \to D^0\gamma$ $(q = u)$	(35.3 ± 0.9) %	< 2.1 MeV	< 10.98
$D_s^{*+} \to D_s^+\gamma$ $(q = s)$	(93.5 ± 0.7) %	< 1.9 MeV	< 16.27

[PDG]

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Let us try to do it using **Light Cone Sum Rules**

Light Cone Sum Rules in a Nutshell

Basic Idea

To calculate the hadronic objects of interest using the analytic properties of the correlation function involved.

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- Uses the unitarity and analyticity of the correlation function.
- Can be written directly in terms of hadronic states.

Ways to calculate a correlation function

Perturbative QCD

- Uses the theory of quarks and gluons.
- Treated in the framework of Light Cone Operator Product Expansion (OPE).

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Matching the two gives estimates for the hadronic objects

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- **Quark Hadron Duality**
  (Relates the non-perturbative spectral function to the perturbatively calculated correlation function)
- **Borel Transformation**
  (To suppress the effect of continuum and higher resonances to reduce the uncertainty due to duality approximation)

Coupling in LCSR

- The correlation function involved:

$$T_{\mu\nu} = -ie\int d^4x e^{ik.x}\langle D_q(p)|T\left\{j_\mu^{em}(x)j_\nu^{D_q^*}(0)\right\}|0\rangle$$

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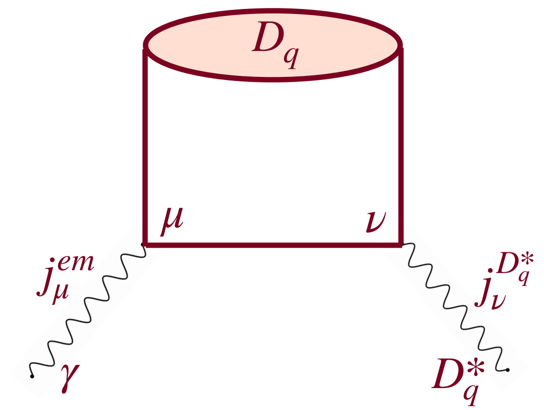

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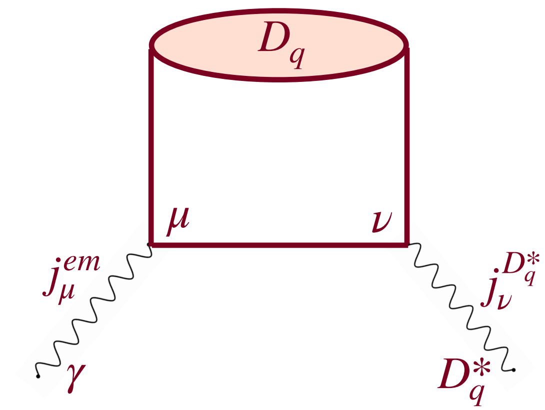

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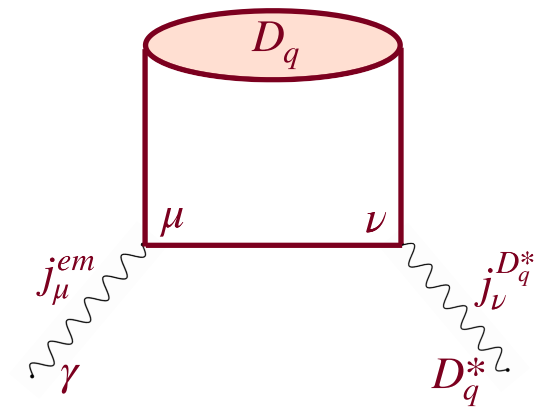
- Bi-quark operator between vacuum & D-state written in terms of D-meson DAs as:

$$\langle D(p) | \bar{c}_\alpha(0)[0,x] q_\beta(x) | 0 > = \frac{i f_D m_D}{4} \int_0^\infty d\omega \, e^{i\omega v.x} \left[(1 + v^\mu \gamma_\mu) \left\{ \phi_+^D(\omega) - \frac{\phi_+^D(\omega) - \phi_-^D(\omega)}{2v\,.\,x} x_\mu \gamma^\mu \right\} \gamma_5 \right]_{\beta\alpha}$$

Momentum of the light quark inside the D-meson

Coupling in LCSR



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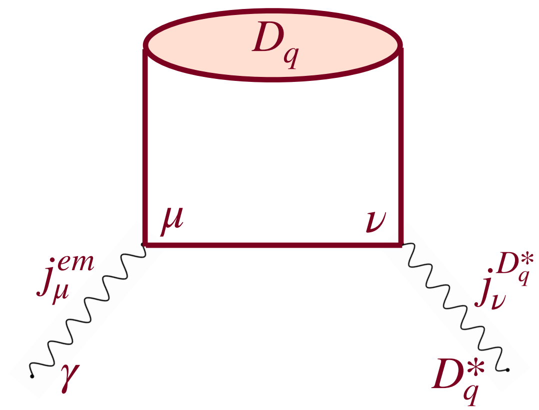

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$$\phi_D^+(\omega) = \frac{\omega}{\omega_0^2}\,\mathrm{Exp}\left[\frac{-\omega}{\omega_0}\right], \qquad \phi_D^-(\omega) = \frac{1}{\omega_0}\,\mathrm{Exp}\left[\frac{-\omega}{\omega_0}\right]$$

→ Exponential Model

[Grozin and Neubert, PRD 55 (1997) 272-290]

$\lambda_D = \omega_0$ in the exponential model (leading order) $\Longrightarrow$ our objective is to find out ω_0

Dispersion Relation

- Using unitarity:

$$T^{had}_{\mu\nu}(p,k) = ie\langle D_q(p)|T\left\{j^{em}_\mu(x)j^{D^*_q}_\nu(0)\right\}|0\rangle \sim \langle D_q(p)|j^{em}_\mu|D^*_q(p+k)\rangle\langle D^*_q(p+k)|j^{D^*_q}_\nu(0)|0\rangle + \sum_n \langle D_q(p)|j^{em}_\mu|n\rangle\langle n|j^{D^*_q}_\nu(0)|0\rangle$$

$$= \frac{2}{m_{D_q}+m_{D^*_q}}\epsilon_{\mu\rho\alpha\beta}\epsilon^{D^*_q}_\rho p_\alpha k_\beta\, G_{D^*_q D_q}(Q^2)$$

$$= f_{D^*_q} m_{D^*_q}\epsilon^{D^*_q}_\nu$$

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$$g_{D_q} = \frac{2}{m_{D_q} + m_{D^*_q}} G_{D_q D^*_q}(Q^2 = 0)$$

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$$\langle D_q(p)|j^{em}_\mu|D_q^*(p+k)\rangle = \frac{2}{m_{D_q}+m_{D_q^*}}\epsilon_{\mu\rho\alpha\beta}\epsilon^{D_q^*}_\rho p_\alpha k_\beta G_{D_q^* D_q}(Q^2)$$

$G_{D_q^* D_q}(Q^2)$: $D_q^* \to D_q$ transition coupling

$$\langle D_q^*(p+k)|j^{D_q^*}_\nu(0)|0\rangle = f_{D_q^*} m_{D_q^*}\epsilon^{D_q^*}_\nu$$

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- The dispersion relation for $G_{D_q^* D_q}$:

$$T^{had}(p,k) = 2e\frac{f_{D_q^*}m_{D_q^*}}{m_{D_q}+m_{D_q^*}}\frac{G_{D_q D_q^*}(Q^2)}{(p+k)^2 - m^2_{D_q^*}} + \int_{m^2_{D_q^*}}^{\infty} ds \frac{1}{\pi}\frac{\text{Im}\left(T^{had}(s,Q^2)\right)}{s-(p+k)^2}$$

The integral term can be approximated using Quark-Hadron duality:

$$= \int_{s_0}^{\infty} ds \frac{1}{\pi}\frac{\text{Im}\left(T^{QCD}(s,Q^2)\right)}{s-(p+k)^2}$$

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- Finally performing the Borel Transformation:

$$\mathcal{B}_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{\exp(-m^2/M^2)}{M^{2(k-1)}}$$

The final SUM RULE

$$G_{D_q^* D_q}(-k^2) = \frac{1}{f_{D_q^*} m_{D_q^*}} \int_0^{s_0} ds\, e^{\frac{\left(m_{D_q^*}^2 - s\right)}{M^2}} \frac{1}{\pi} \mathbf{Im}\left(T^{QCD}(s, Q^2)\right)$$

$$T^{QCD}(s, Q^2) = e f_{D_q} m_{D_q} \int_0^{\infty} dw \left\{ \phi_+^D(w) \left[\frac{Q_C}{(k - wv)^2 - m_c^2} + \frac{Q_q}{(k + wv)^2 - m_q^2} \right] + \psi_\pm^D(w) \left[\frac{Q_C m_c}{\left((k - wv)^2 - m_c^2\right)^2} + \frac{Q_q m_q}{\left((k + wv)^2 - m_q^2\right)^2} \right] \right\}$$

$$\Psi_\pm^D(\omega) = \int_0^{\omega} d\tau \left(\phi_+^D(\omega) - \phi_-^D(\omega)\right)$$

M : The Borel Mass
s_0 : The continuum threshold

→ The independent parameters.

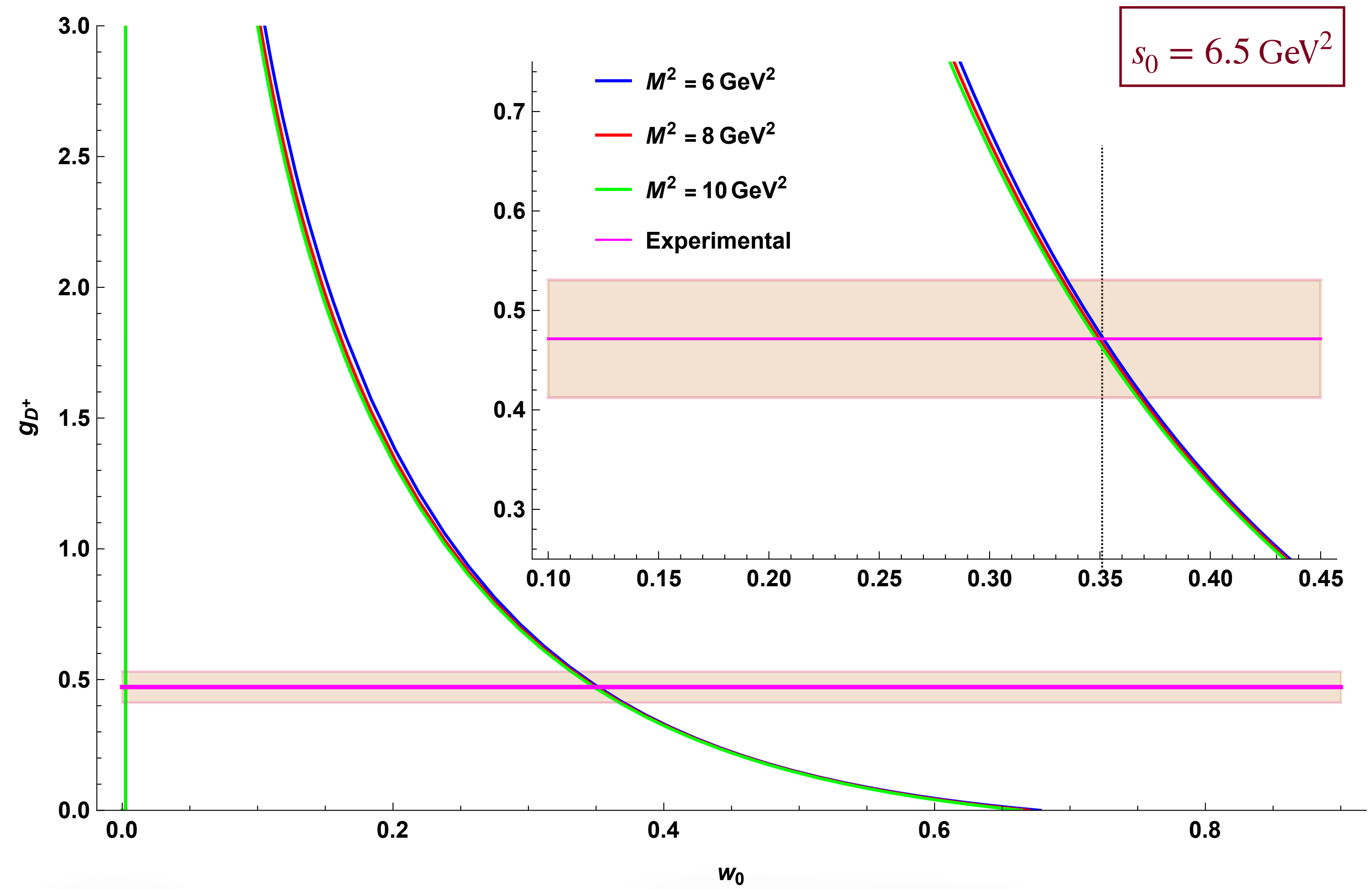
→ To be fixed by demanding that the sum rule should be saturated by the lowest state and the contributions coming from the heavier and continuum states are suppressed.

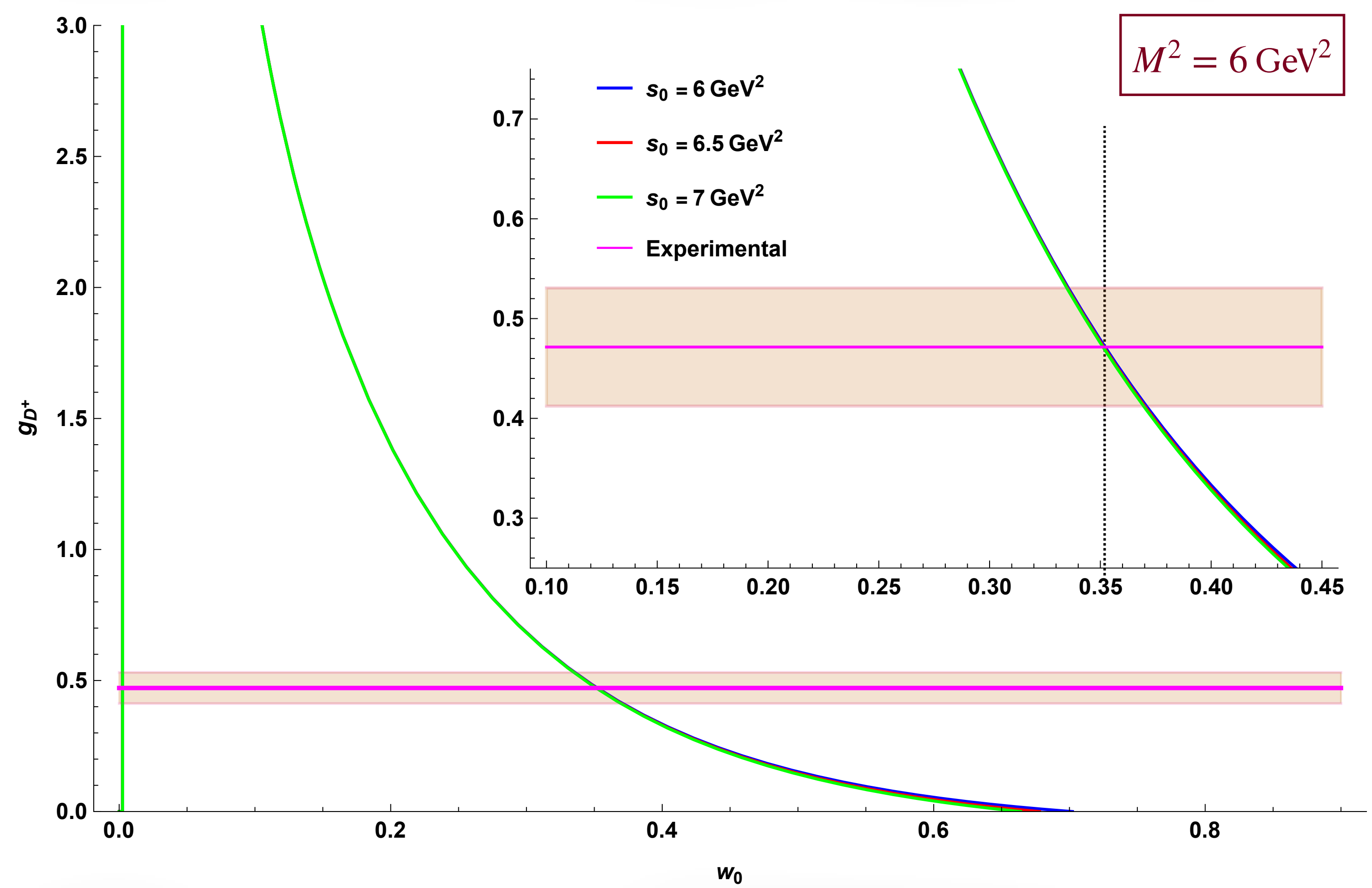
Results (Preliminary)

- The analytic results for g_{D_q} is consistent with the heavy quark and chiral symmetry prediction, according to which $g_{D_q} \sim \dfrac{Q_c}{m_c}$.

  [Amundson et. Al, PLB 296 (1992) 415-419]

- Matching the LCSR result with experimental data for $g_{D^+} \Longrightarrow \omega_0 \sim 0.35 \pm 0.02$ GeV.





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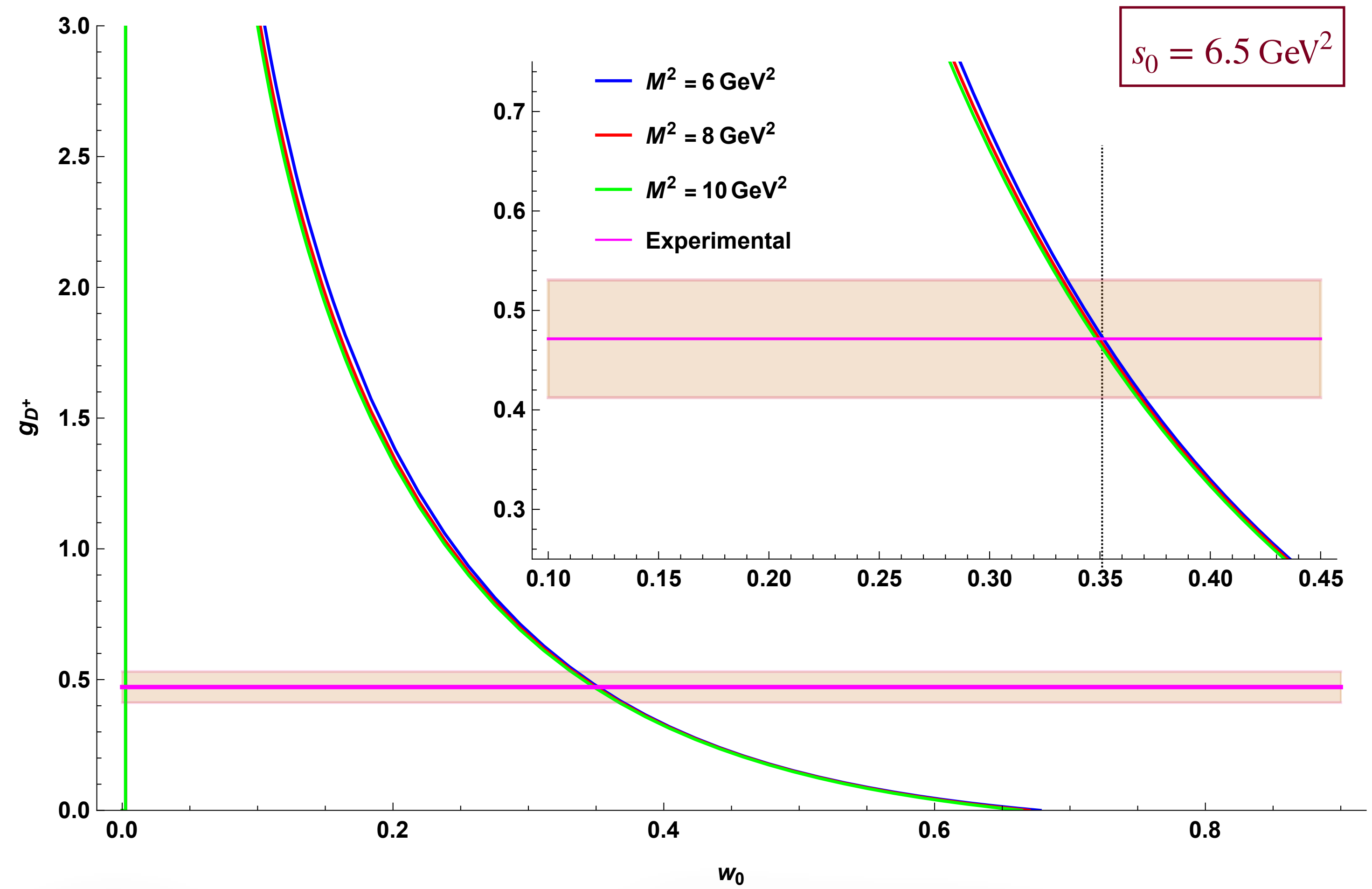
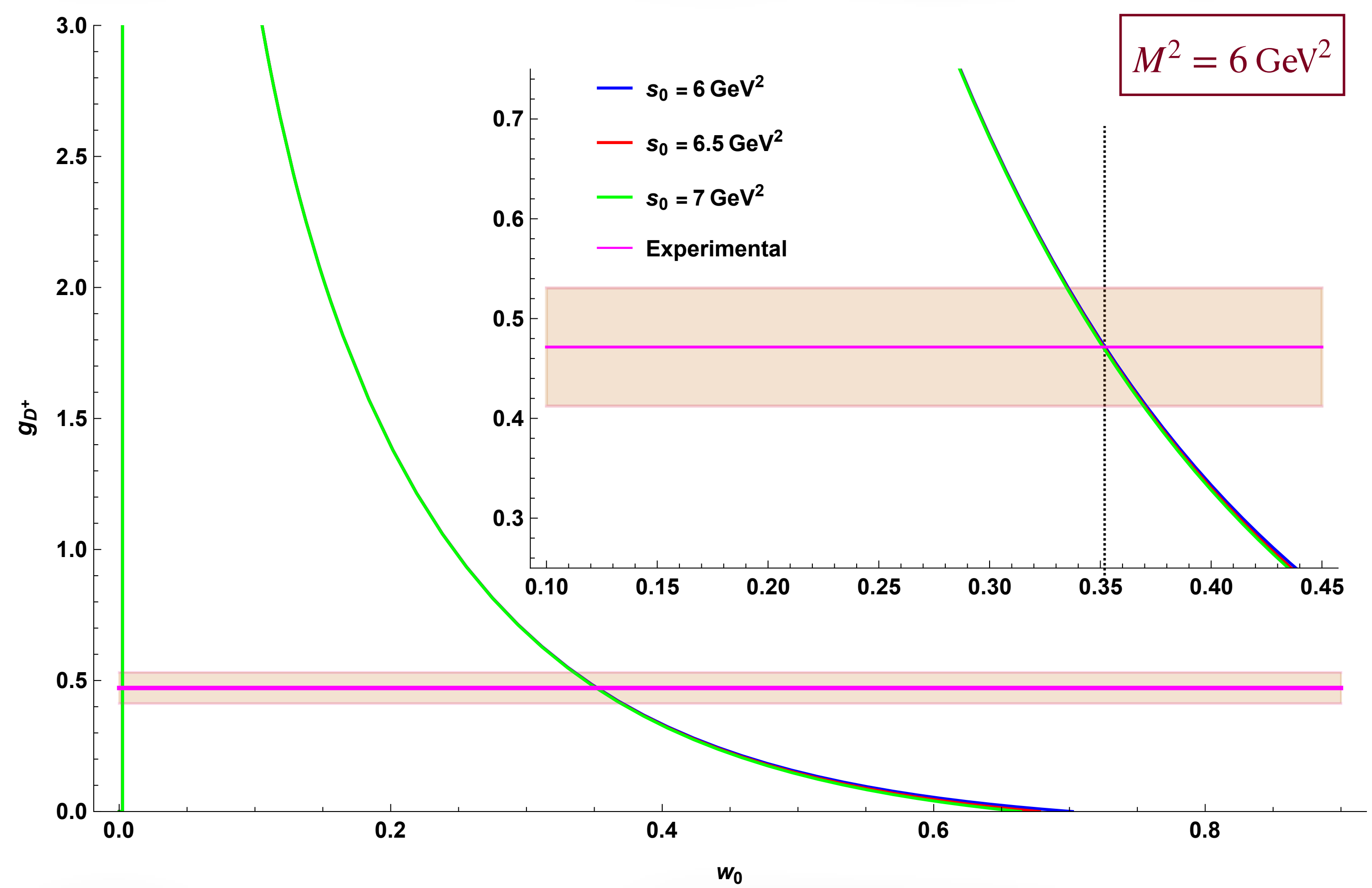
- Corrections coming from higher order effects and the RG evolution of DAs may play very crucial role (underway.)





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  [Amundson et. Al, PLB 296 (1992) 415-419]

- Matching the LCSR result with experimental data for $g_{D^+} \Longrightarrow \omega_0 \sim 0.35 \pm 0.02$ GeV.

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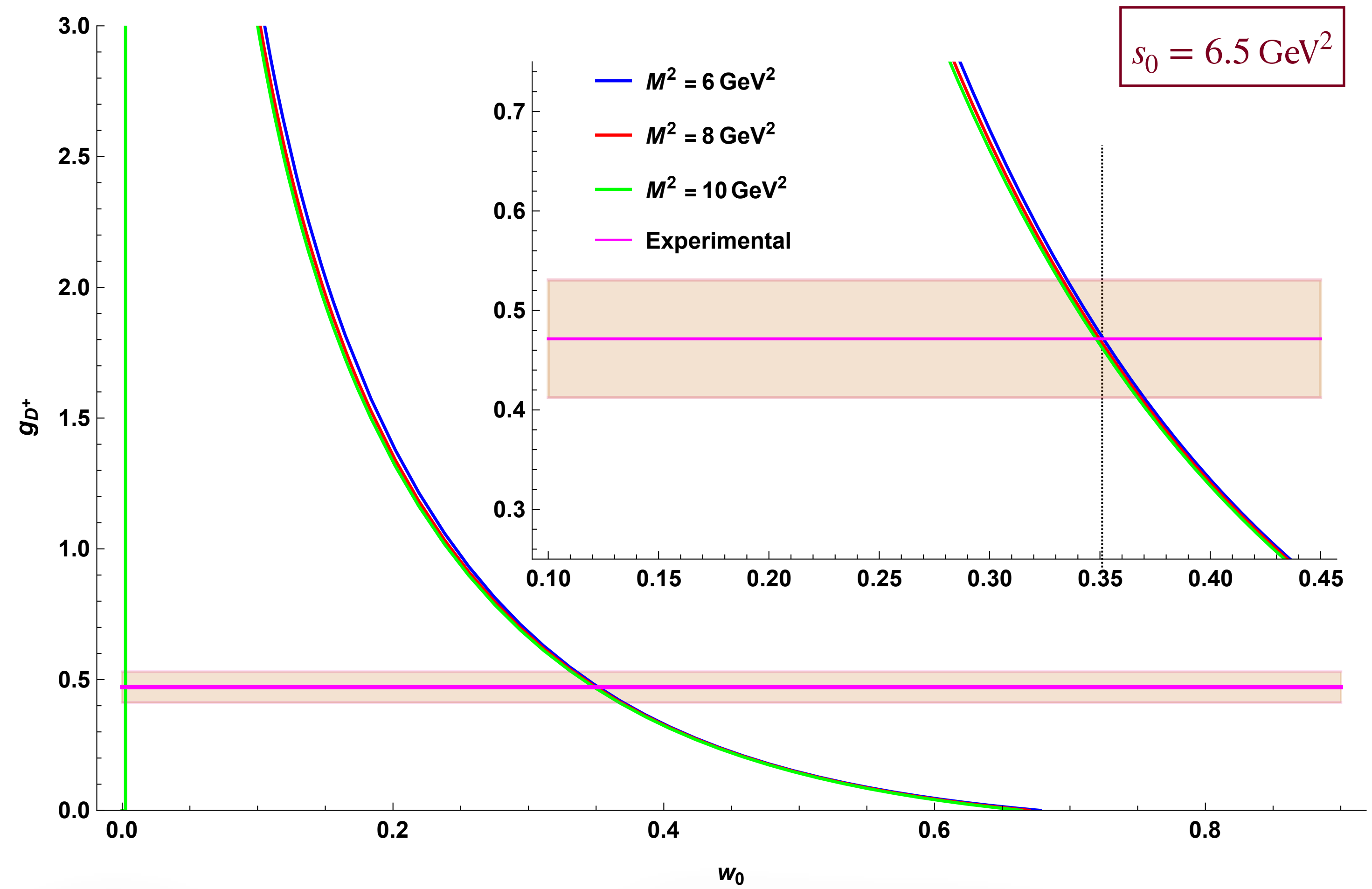

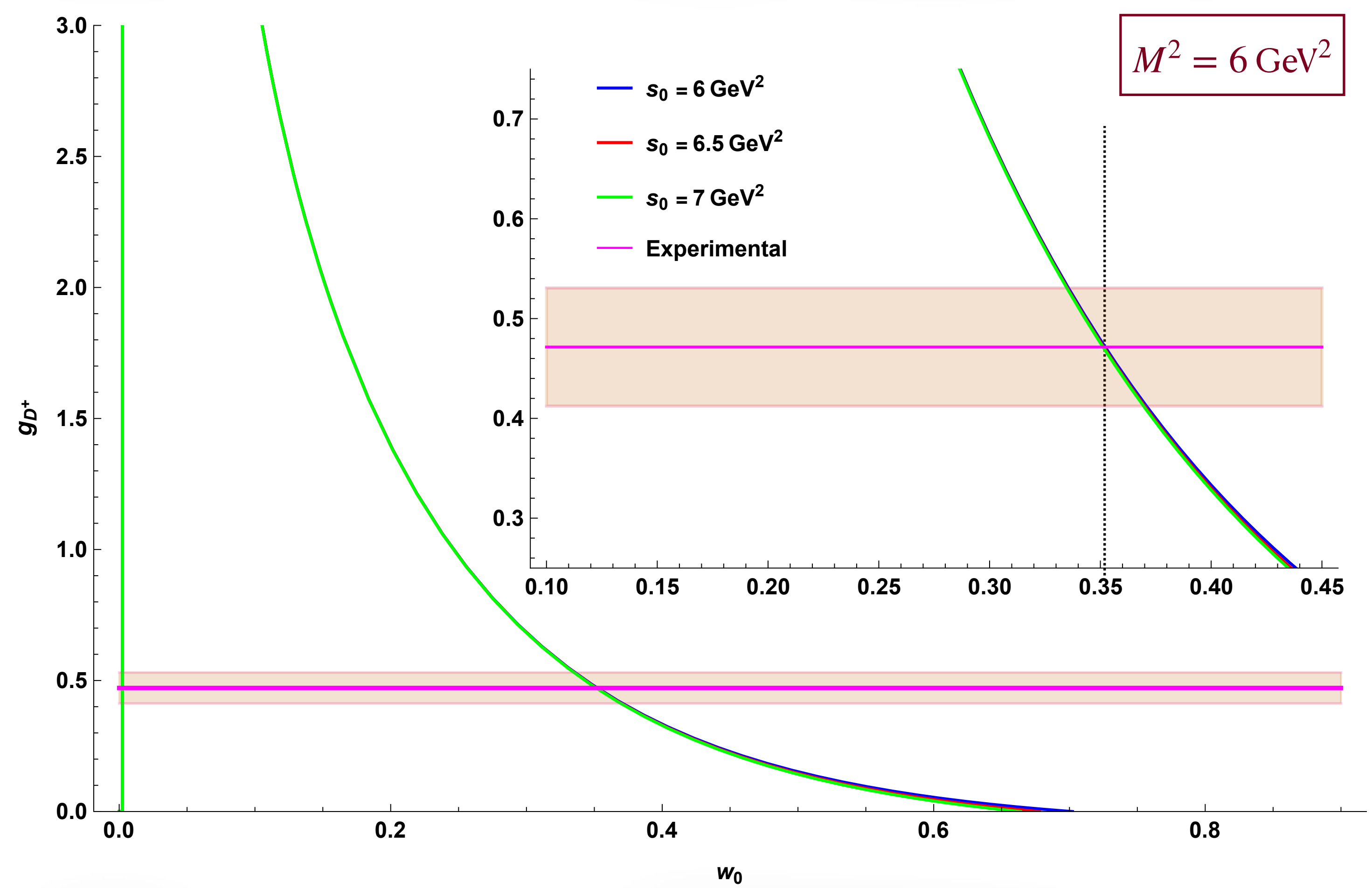

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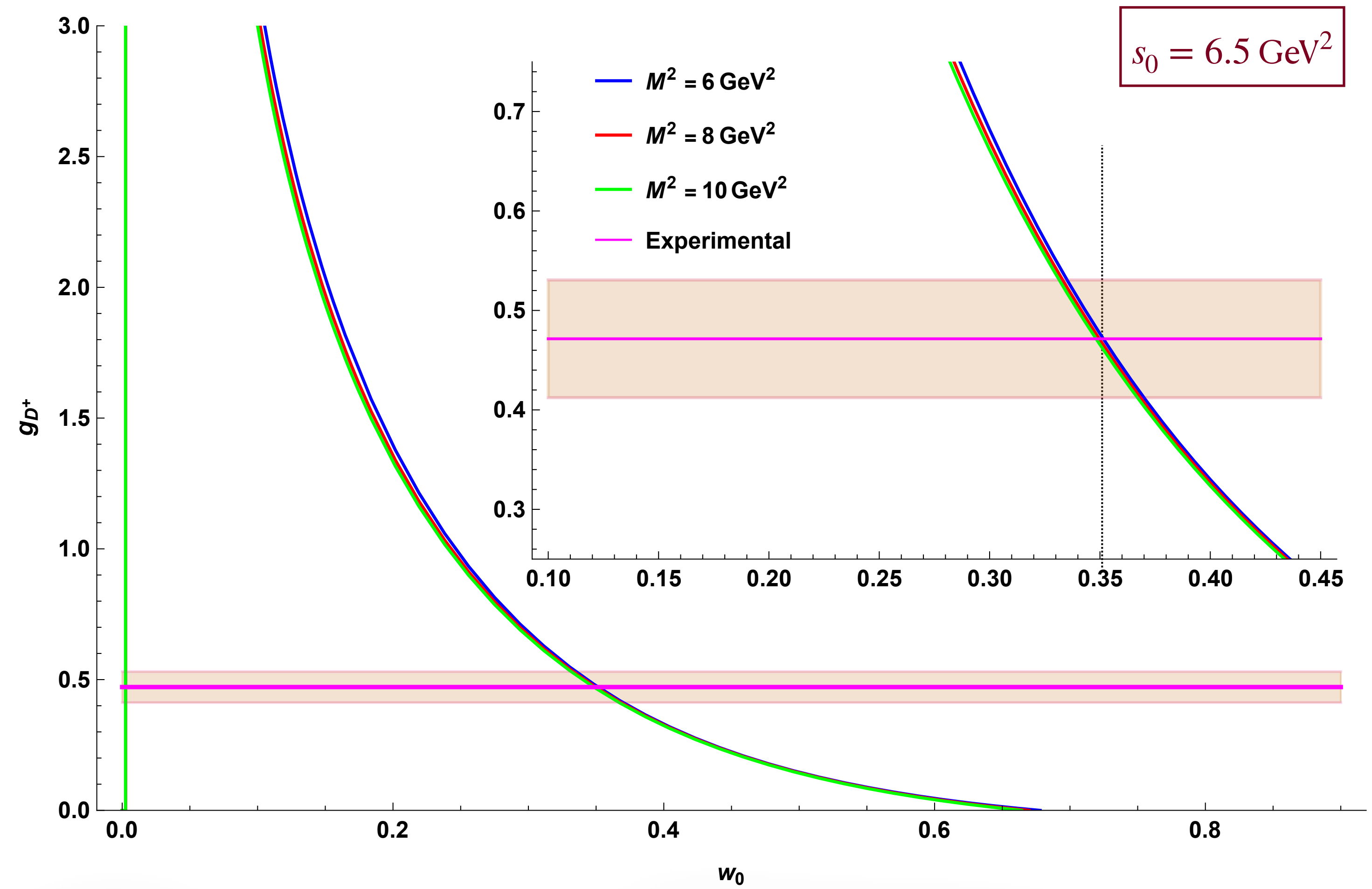
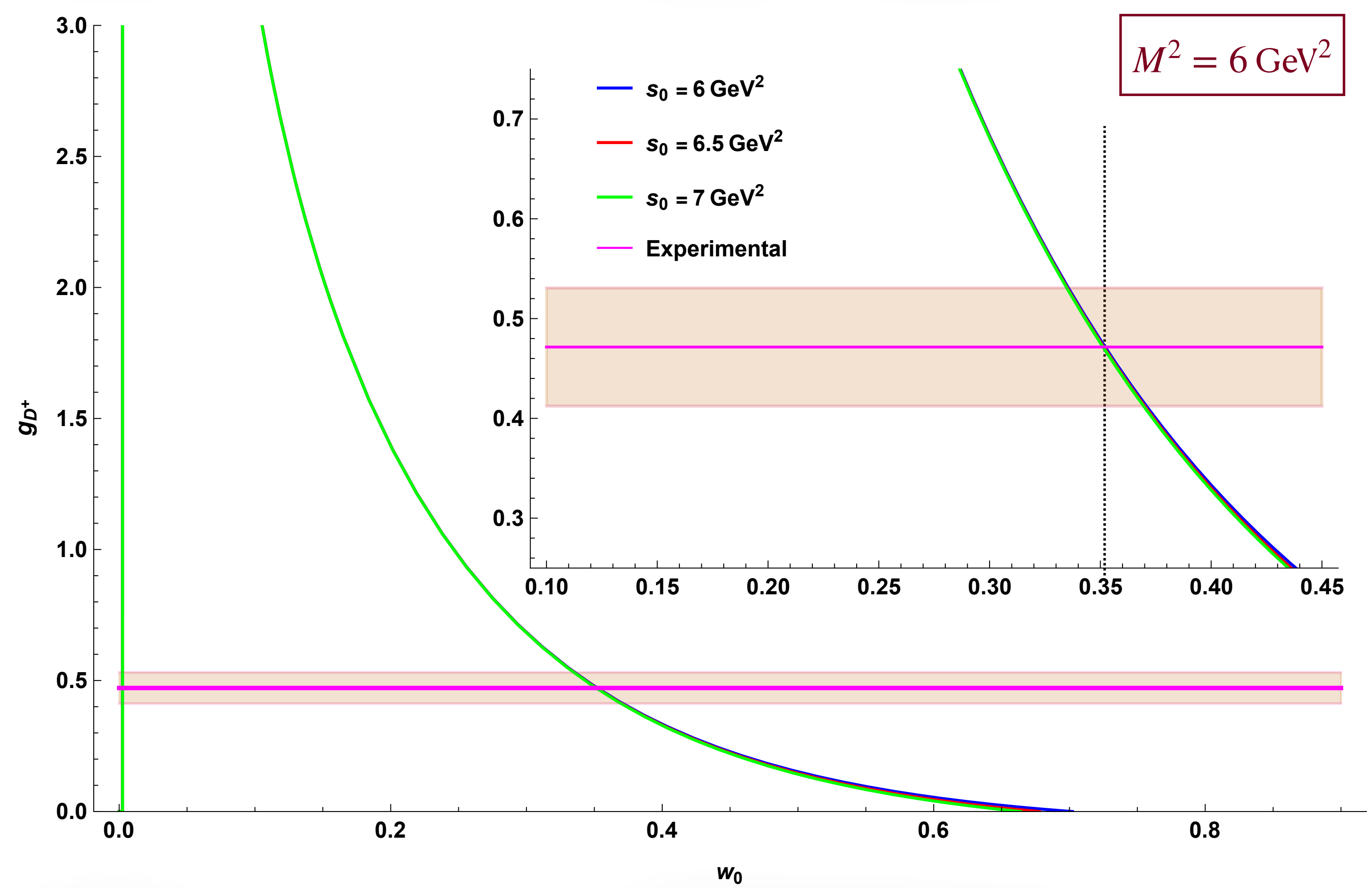
- The difference or similarity between the ω_0 values for different mesons is expected to shed some light on the SU(3) violation effects in the charm system.





Summary and Outlook

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- A similar analysis will be helpful in getting estimates for B-meson DAs (less corrections expected due to higher order effects) : Experimental data is missing.
- Proper estimates of the total decay width of the vector heavy mesons and their radiative decay channels are required at the experiments.

Questions/ Comments?